ETTORE VITTONE

Dipartimento di Fisica, Università di Torino

Modeling of damage in ion irradiated semiconductors

Radiation damage is the general alteration of the operational properties of a semiconductor devices induced by ionizing radiation

Three main types of effects:

- Transient ionization. This effect produces electron-hole pairs; particle detection with semiconductors is based on this effect.

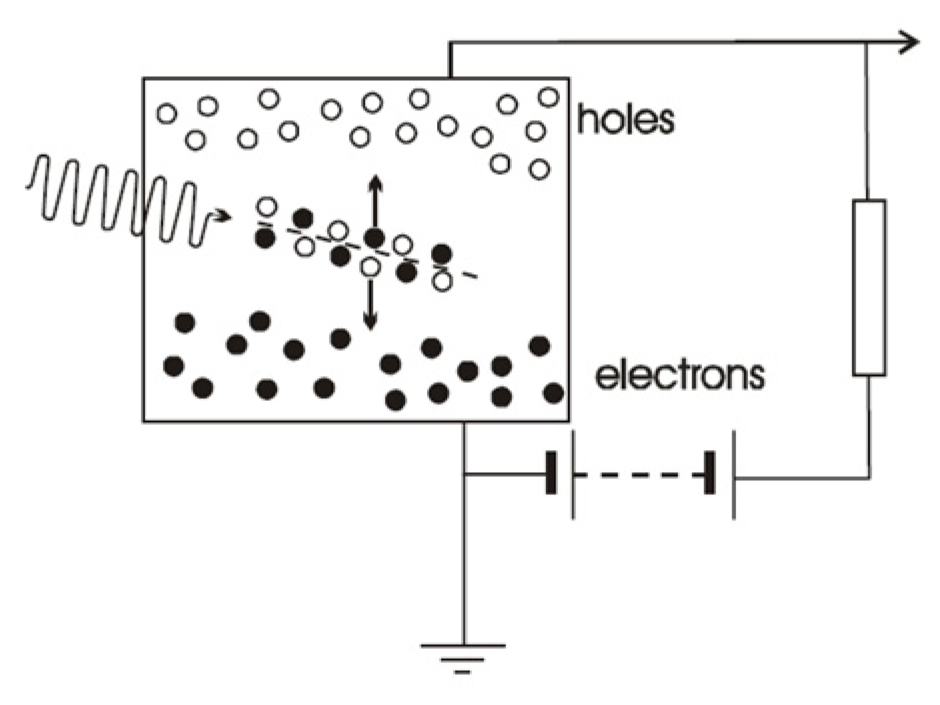
-Long term ionization. In insulators, the material does not return to its initial state, if the electrons and holes produced are fixed, and charged regions are induced.

- Displacements. These are dislocations of atoms from their normal sites in the lattice, producing less ordered structures, with long term effects on semiconductor properties.

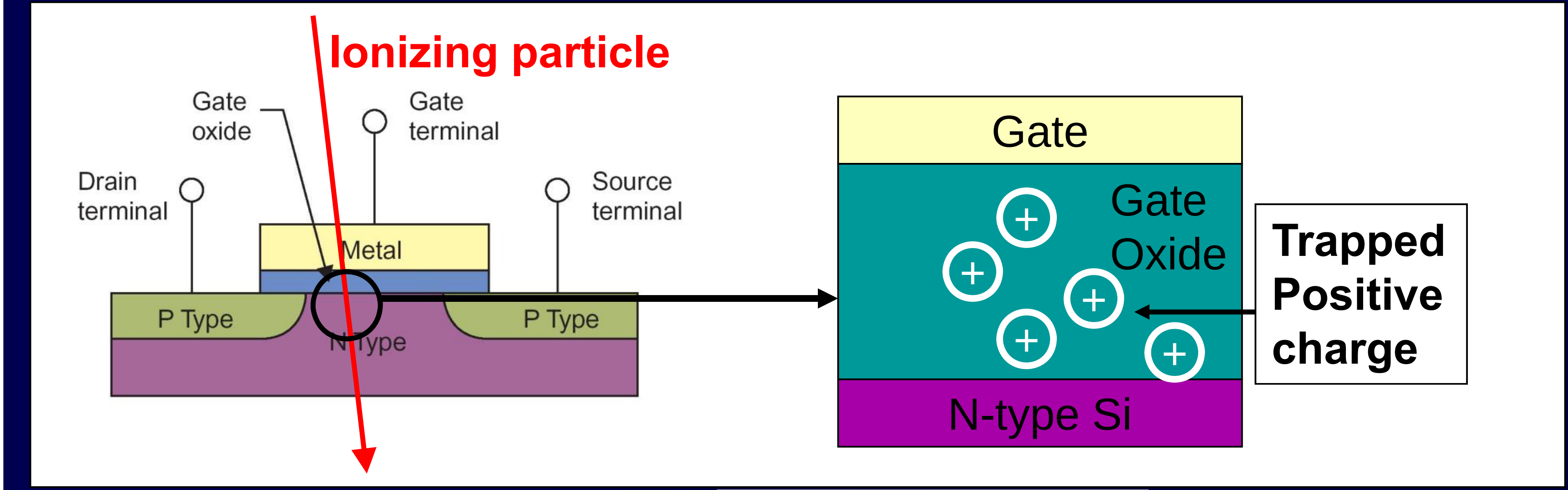
V.A.J. van Lint, The physics of radiation damage in particle detectors, Nucl. Instrum. Meth. A253 (1987) 453.

Transient ionization.

This effect produces electron-hole pairs; particle detection with semiconductors is based on this effect. (IBIC)



-Long term ionization. In insulators, the material does not return to its initial state, if the electrons and holes produced are fixed, and charged regions are induced.



- **Parametric shifts** in transistors parameters due to the build-up of trapped positive charge and interface states caused by several low-LET particles striking a chip
- Total Ionizing Dose affects **dielectric layers** (e.g., gate oxide, isolation oxides)

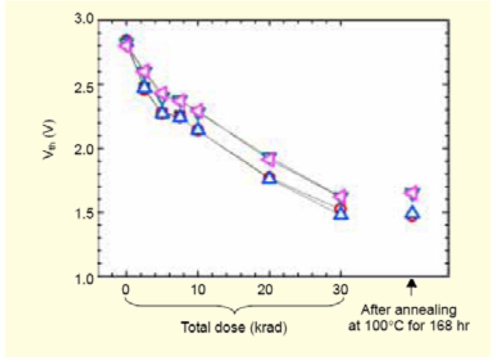


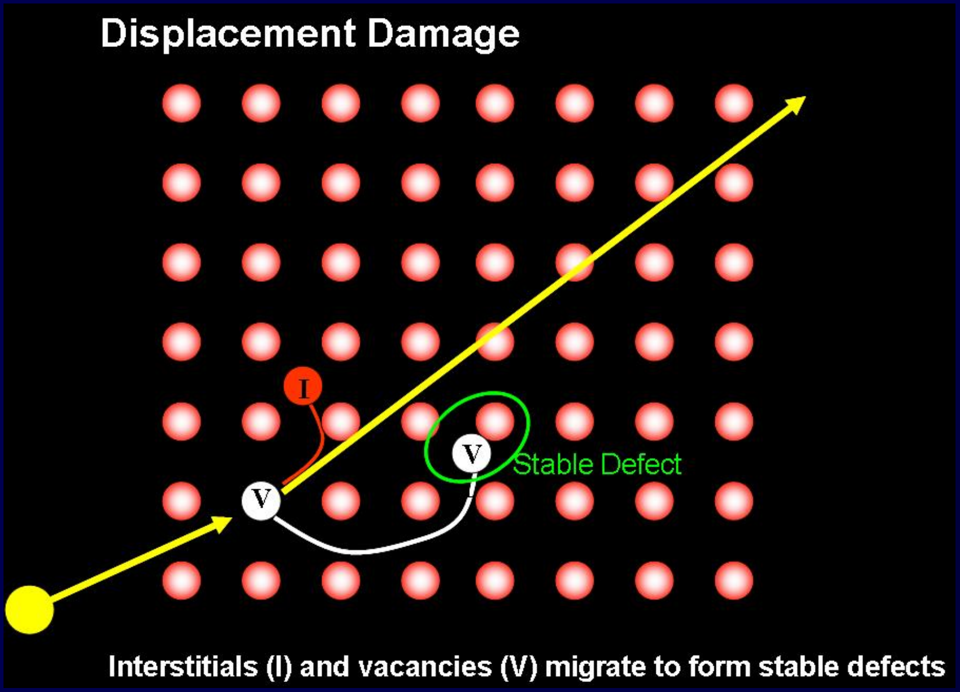
Fig. 1. Threshold voltage (V_{th}) characteristics of a MOSFET (200 V) under a dose rate of 9.55 rad/s.

Young Hwan Lho, Ki Yup Kim
Radiation Effects on the Power MOSFET for space applications

http://etrij.etri.re.kr/Cyber/Download/PublishedPaper/2704/S27-04-14.pdf

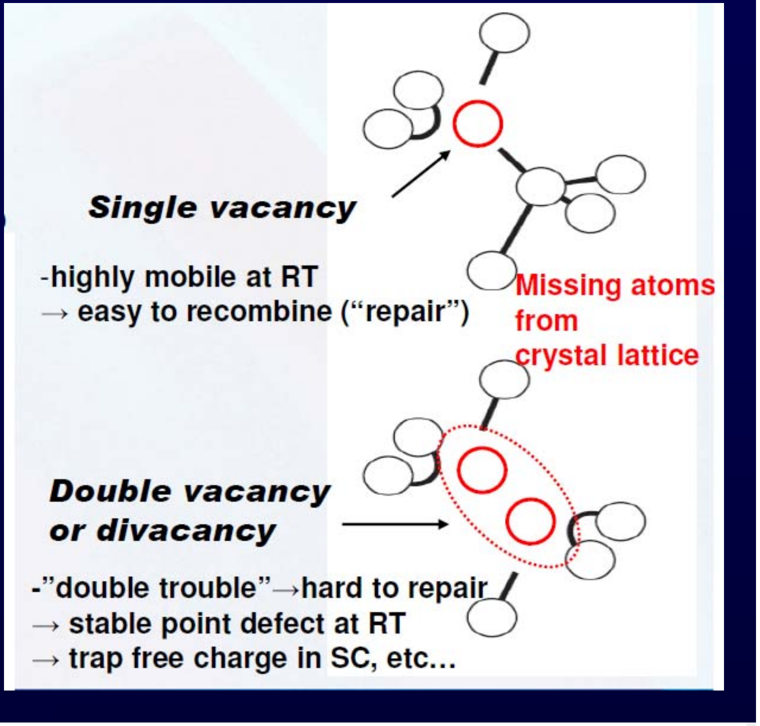
G. Vizkelethy, "*radiation effects in microelectronic devices*", Thursday 9-10.30

- Displacements. These are dislocations of atoms from their normal sites in the lattice, producing less ordered structures, with long term effects on semiconductor properties



Displacement Damage

Interstitials (I) and vacancies (V) migrate to form stable defects



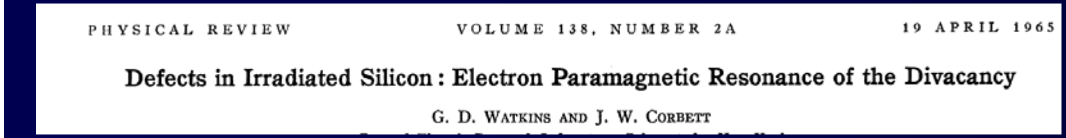
Single vacancy

-highly mobile at RT
→ easy to recombine (“repair”)

Missing atoms from crystal lattice

Double vacancy or divacancy

-”double trouble”→hard to repair
→ stable point defect at RT
→ trap free charge in SC, etc…



PHYSICAL REVIEW VOLUME 138, NUMBER 2A 19 APRIL 1965

Defects in Irradiated Silicon: Electron Paramagnetic Resonance of the Divacancy

G. D. Watkins and J. W. Corbett

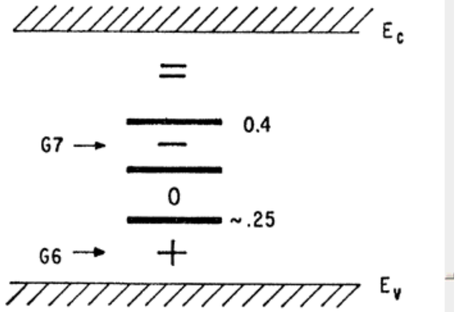
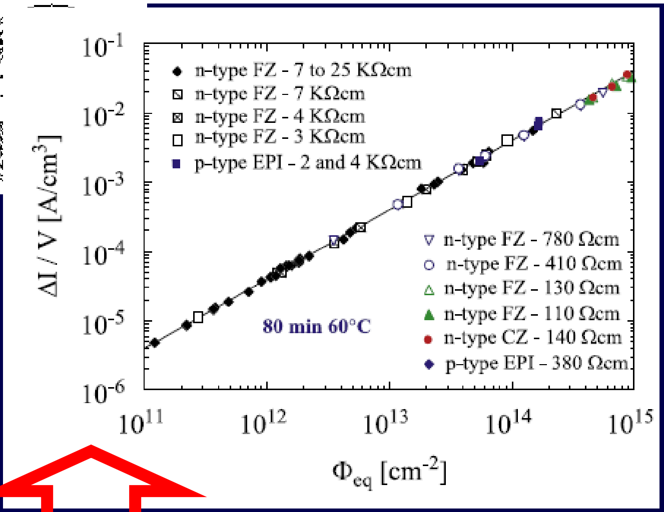
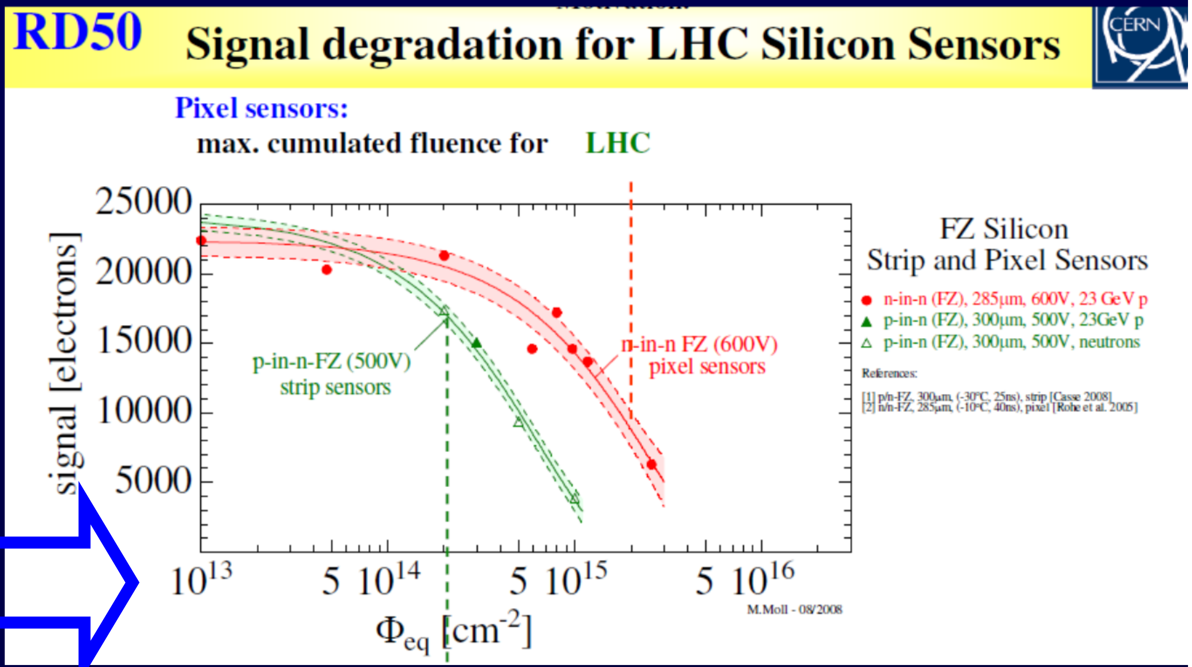


FIG. 14. Electrical levels associated with the divacancy. The level positions (in eV) are given to the nearest band edge. The charge states giving rise to the G6 and G7 spectra are indicated.

http://holbert.faculty.asu.edu/eee560/RadiationEffectsDamage.pdf

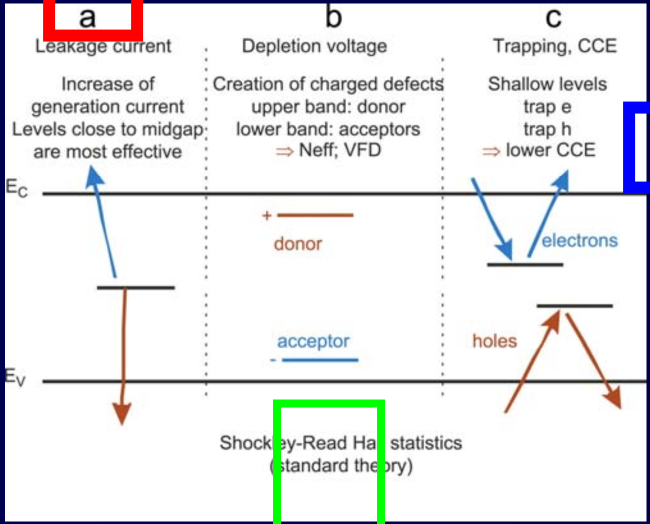
M. Bruzzi, M. Moll, RD50, 2010
http://indico.cern.ch/getFile.py/access?contribId=9&sessionId=2&resId=1&materialId=slides&confId=81149

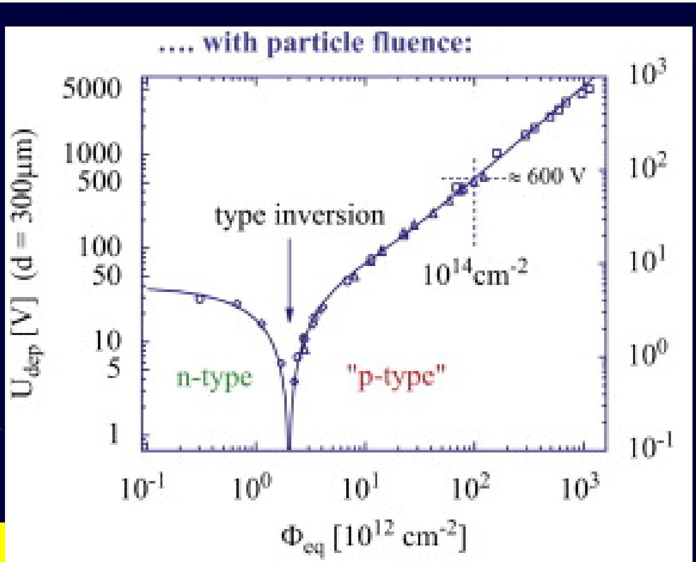




RD50 **Signal degradation for LHC Silicon Sensors**

Pixel sensors:
max. cumulated fluence for LHC





Frank Hartmann, Silicon tracking detectors in high-energy physics
Nuclear Instruments and Methods in Physics Research A 666 (2012) 25–46

Shockley-Read-Hall Model

Applet Recombination

Excess carrier lifetime

$$\tau = \frac{1}{N_{trap} \cdot \sigma \cdot v_{th}}$$

Trap density

Capture cross section

Thermal velocity

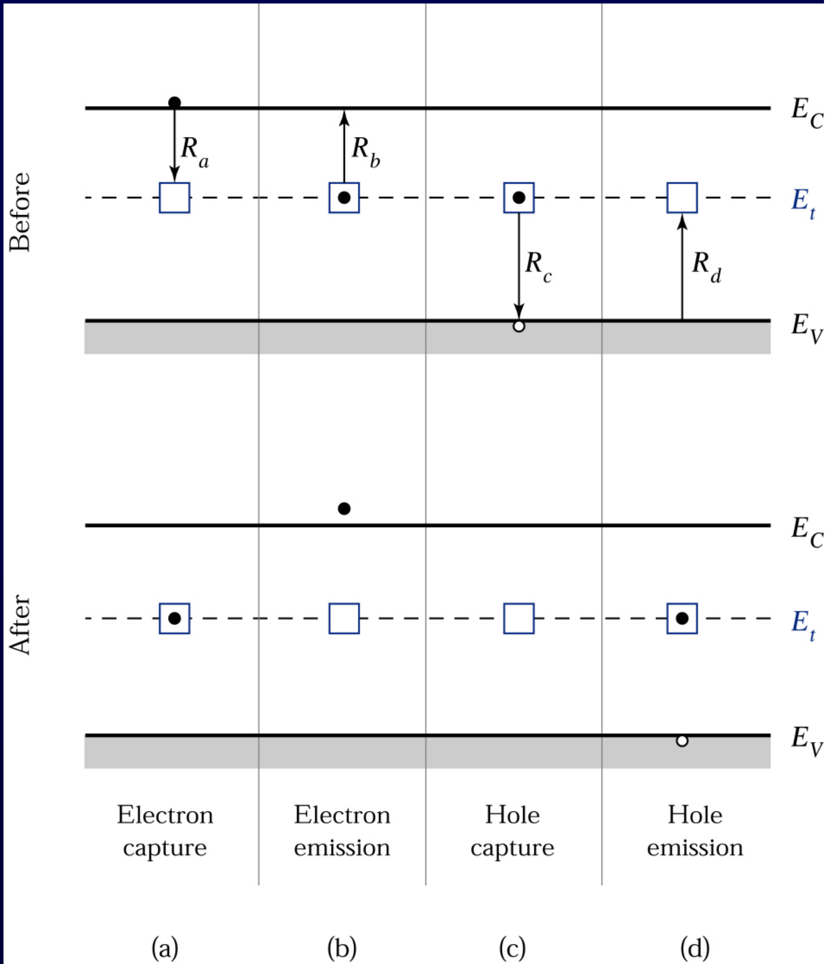


Figure 3.12.
Indirect generation-recombination processes

$$\tau = \frac{1}{N_{trap} \cdot \sigma \cdot v_{th}}$$

Trap density in pristine material

$$N_{trap} = N^{0}_{trap} + k \cdot \Phi$$

Trap density induced by radiation

$$\frac{1}{\tau(\Phi)} = \frac{1}{\tau_0} + K \cdot \Phi$$

Modeling radiation degradation in solar cells extends satellite lifetime

Robert J. Walters, Scott Messenger, Cory Cress, Maria Gonzalez and Serguei Maximenko

A physics-based model of the effect of radiation on the performance of solar cells in space may enhance the on-orbit lifetime of Earth-orbiting spacecraft.

3 January 2011, SPIE Newsroom. DOI: 10.1117/2.1201012.003417

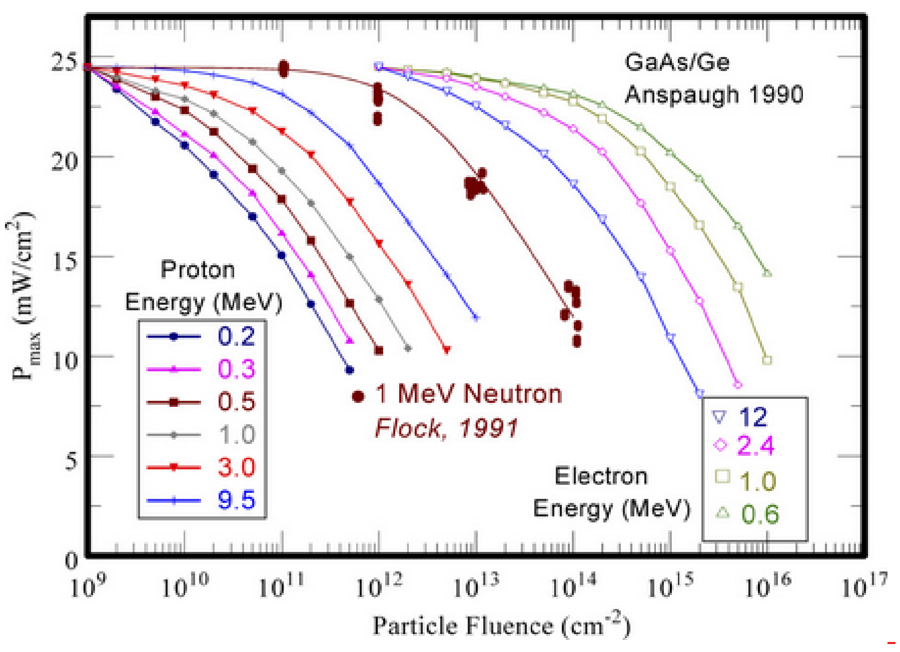


Figure 2.*Measured degradation of a single junction gallium arsenide (GaAs) solar cell under proton, electron,[2]and neutron irradiation.[3] These data can be used to empirically determine the energy dependence of the solar-cell degradation thereby enabling on-orbit performance prediction. P_{max}: Maximum power.*

Space environment-> wide spectrum of ions (protons) and electrons.

To understand the performance of a solar cell in the space radiation environment, it is necessary to know how cell degradation depends on the energy of the irradiating particle.

http://spie.org/x43655.xml

NIEL hypothesis:

the radiation damage is linear proportional to the non-ionizing energy loss of the penetrating particles (radiation) and this energy loss is again linear proportional to the energy used to dislocate lattice atoms (displacement energy).

Final concentration of defects depends only on NIEL and not on the type an initial energy of the particle.

Number of displacements (I-V pairs) is proportional to PKA energy (Kinchin-Pease: N=T/2TD; T: PKA energy; TD: threshold energy to create a Frenkel pair).

Displacement damage dose:

$$D_d = \int \mathrm{NIEL}(E) \cdot \frac{d\Phi}{dE} dE$$

UNITS:

NIEL:(Energy per unit length)/(material density):keV·cm2/g

(in high energy physics the displacement damage cross section (D) in MeV·mb is usually used)

Dd : Energy per unit mass:keV/g

(G.P. Summers et al., IEEE Trans. Nucl. Sci., Vol. 40, pp. 1372, 1993)

How to calculate NIEL from SRIM

10 MeV H+ in
Si 100 μm thick

1. Run SRIM and evalutate the total number of vacancy/ion W
2. Evaluate the energy required to create a vacancy M using the modified Kinchin-Pease relationship: the term 2 is due to the binding energy loss that SRIM assign to each vacancy
   Ed is the displacement energy
3. L is the device length and ρ is the mass density

$$\mathrm{NIEL} = \frac{\mathrm{M} \cdot \mathrm{Vac}_{\mathrm{Tot}}}{\rho \cdot \mathrm{R}}$$

W=4.7 Vac/ion/μm

Ed=20 eV
M=52 eV/vac

$$\mathrm{M} = \left(\frac{\mathrm{E_d}}{0.8} + 2 \right) \mathrm{eV}$$

ρ =2.3 g/cm^3
L=100 μm

S. R. Messenger et al., *Using SRIM to Calculate the Relative Damage Coefficients for Solar Cells*, Prog. Photovolt: Res. Appl. 2005; 13:115–123

If Y is the physical observable (e.g. conductivity, maximum output power for solar cells, Charge Collection Efficiency (CCE) in radiation detectors), which characterizes a tested device subjected to radiation damage, its degradation can be modelled by the following phenomenological relationship:

Device characteristic after irradiation

$$\eta = \frac{Y}{Y_0} = 1 - K \cdot \Phi = 1 - K_{ed} \cdot D_d$$

Device characteristic before irradiation

Particle Fluence

Equivalent damage factor

Displacement dose

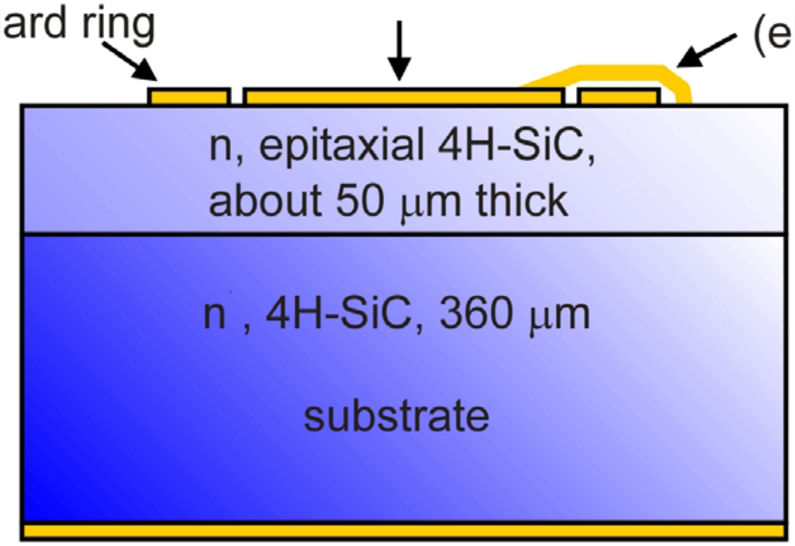
Samples

Starting Material: 360 μm n-type 4H-SiC by CREE (USA)
Epitaxial layer from Institute of Crystal Growth (IKZ), Berlin, Germany
Devices from Alenia Marconi System

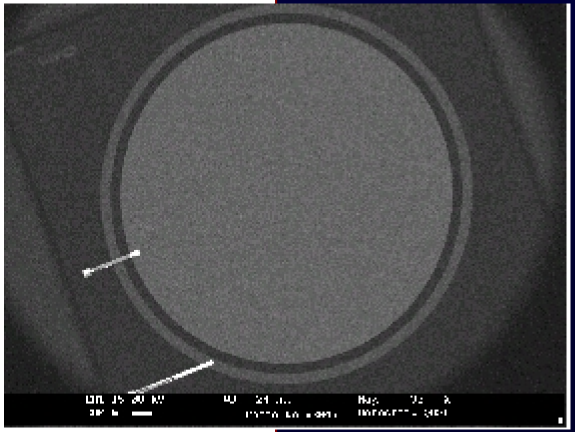
Circular Schottky contact
(100 nm Ni, 100 nm Au)
Ø = 5 mm or Ø = 3 mm

Guard ring

Air-bridge
(electroplated Au, 3 μm)



Ohmic contact, Ti/Pt/Au



EXPERIMENTAL PROCEDURE:

Nuclear microprobe facility @ Ruđer Bošković Institute (Zagreb)

Irradiation of an area of 5400 μm^2 by 2 MeV and 1.5 MeV protons.

Final Fluence:

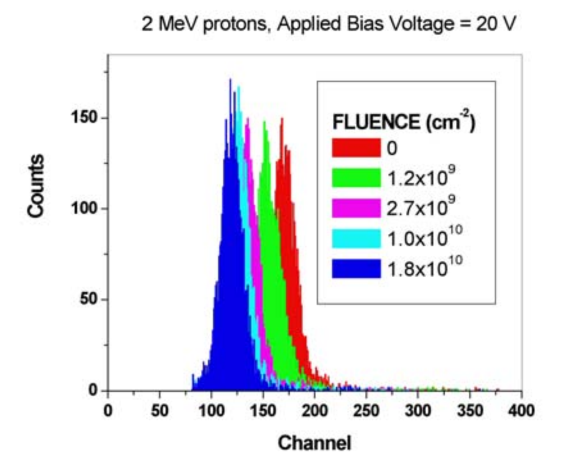
1.2×10^6 protons/ $(68 \times 79) \mu m^2 \approx 2 \times 10^{10}$ protons/cm^2

Applied bias voltage = 20 V, 40 V,60 V,...120 V

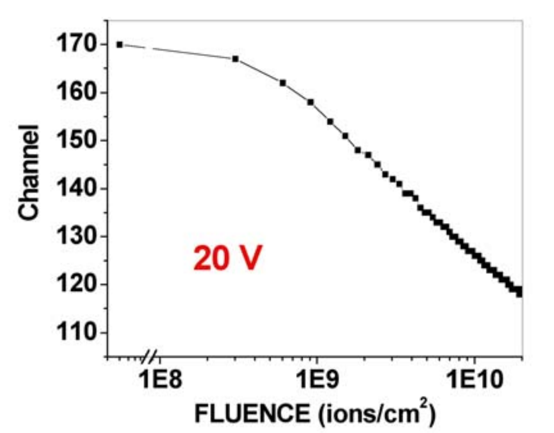
Event by event data acquisition mode.

OFF LINE ANALYSIS

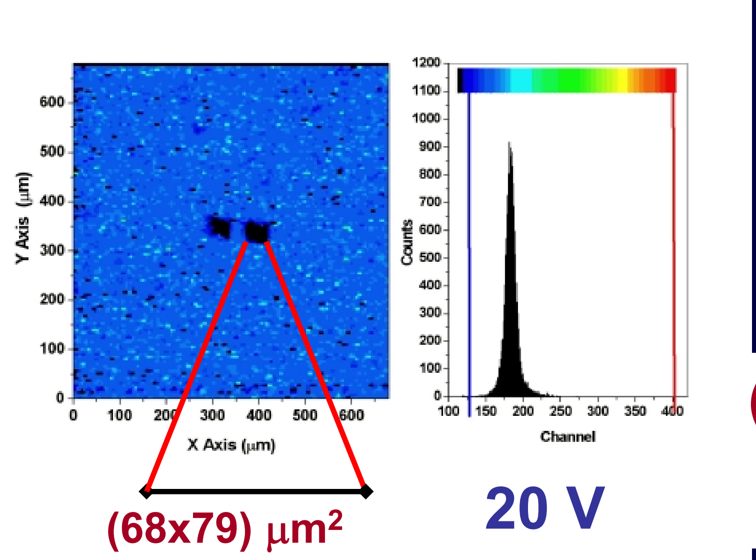
For each scan (about 10^8 ions/cm^2), pulse height spectra are recorded



The median pulse height is evaluated as a function of ion fluence



CONTROL

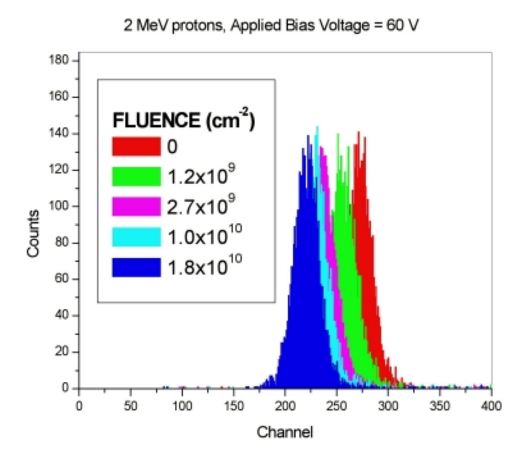


(650x650) µm²

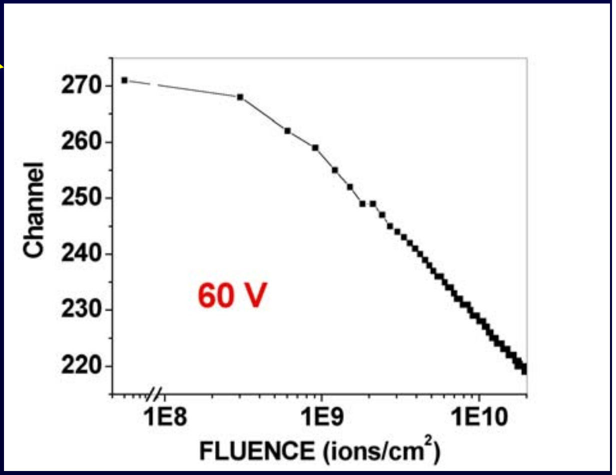
IBIC map

OFF LINE ANALYSIS

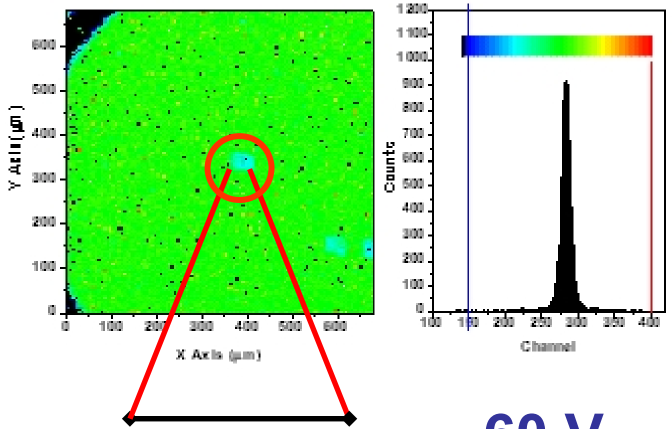
For each scan (about 10^8 ions/cm^2), pulse height spectra are recorded



The median pulse height is evaluated as a function of ion fluence



CONTROL



(68x79) µm^2

60 V

(650x650) µm^2

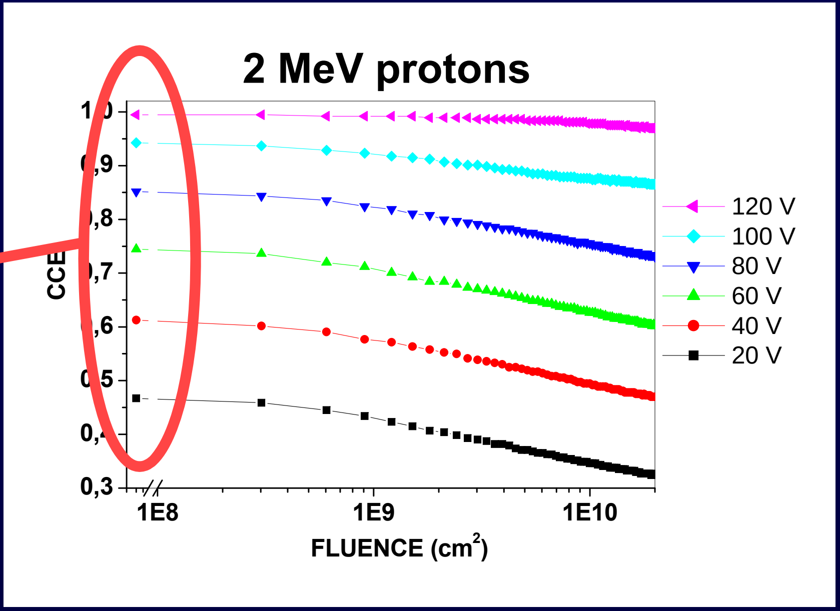
IBIC map

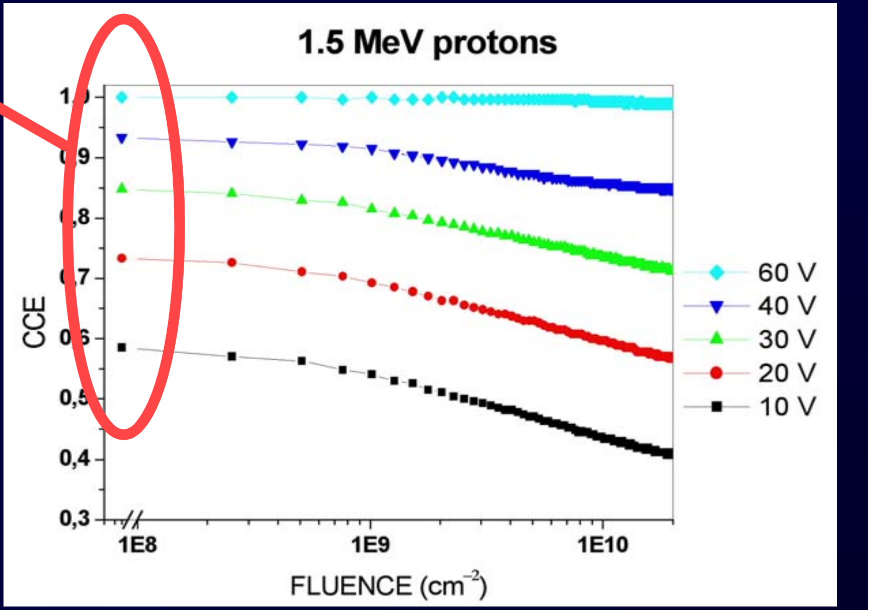
Charge collection efficiency

Vs.

Ion Fluence

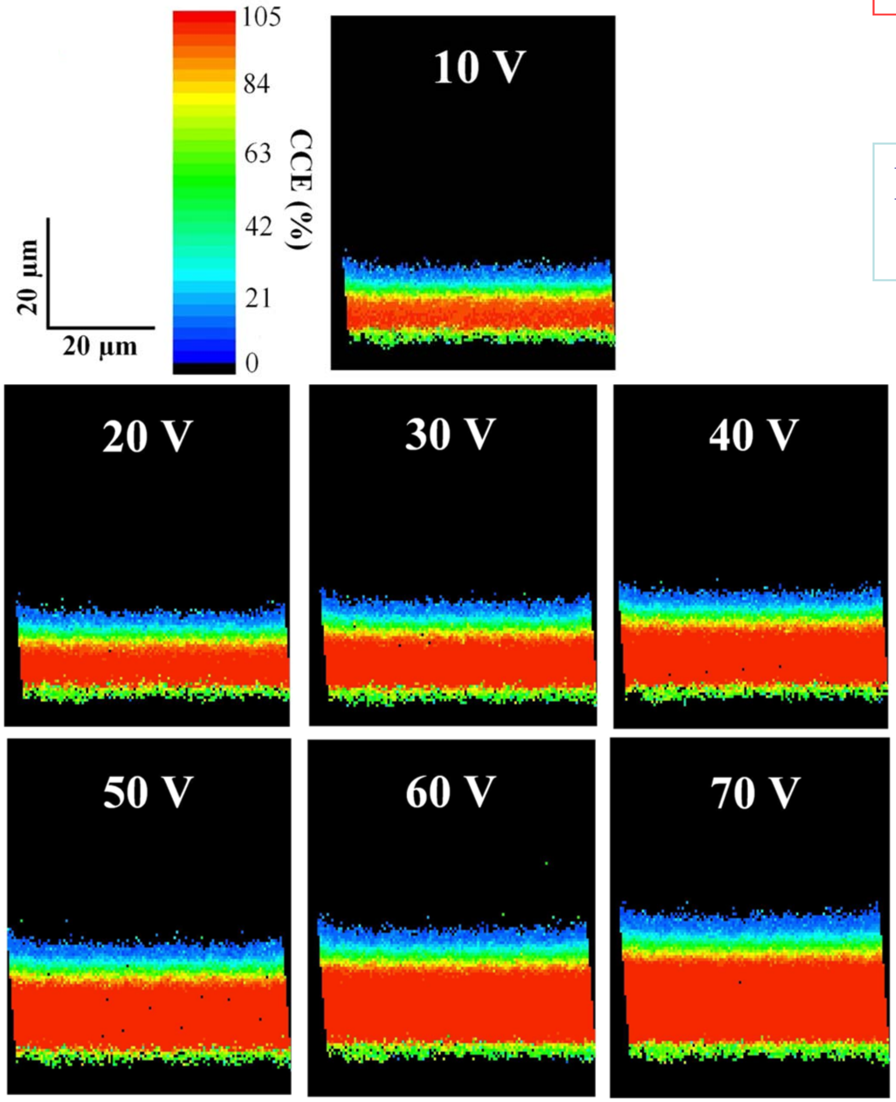
PRISTINE CONDITIONS



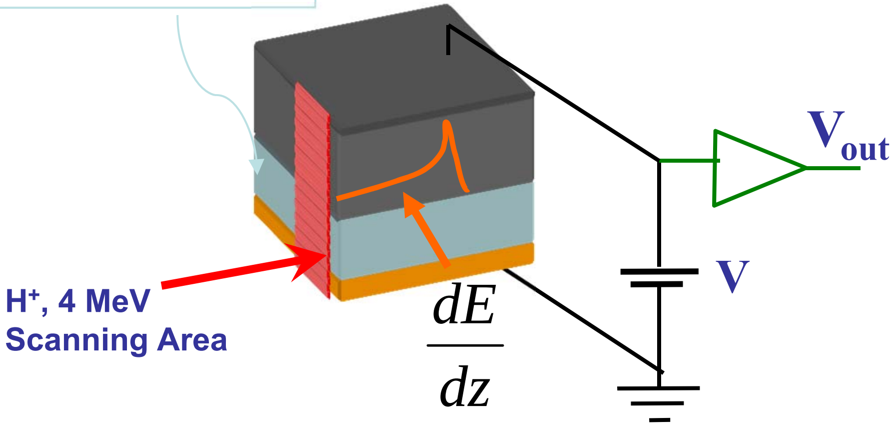


Lateral IBIC

Protons, 4MeV on SiC

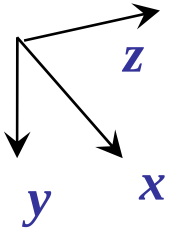


Depletion Region
w **(V)**



H⁺, 4 MeV
Scanning Area

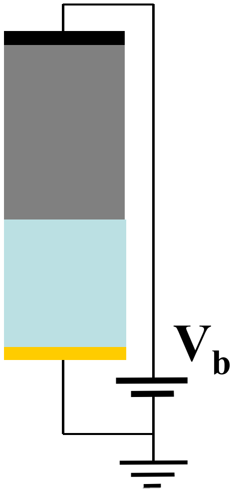
Range = 100 µm

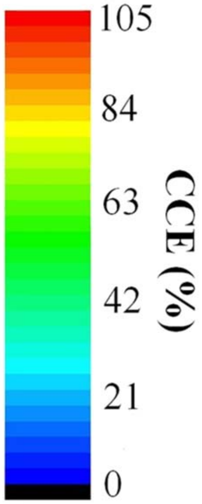


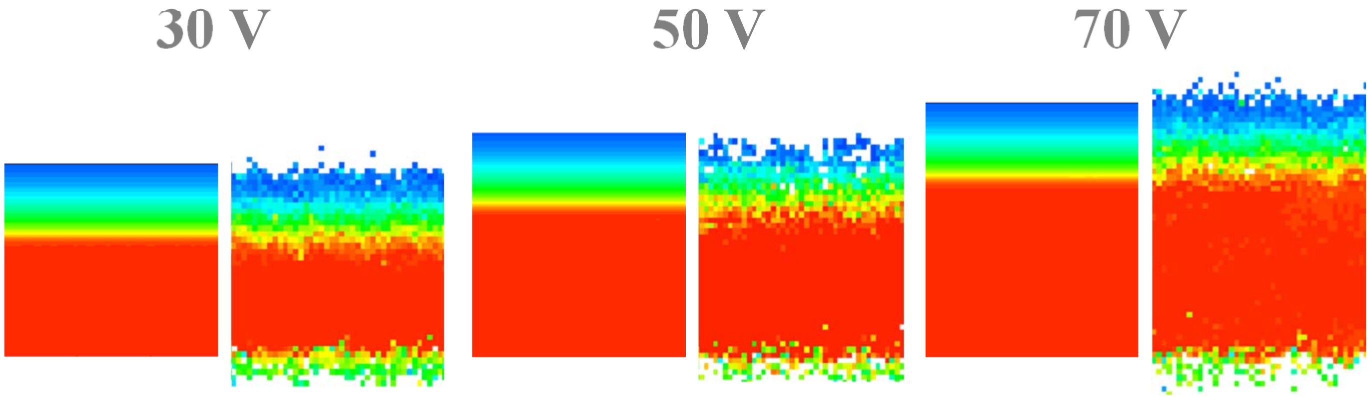
Numerical Simulations

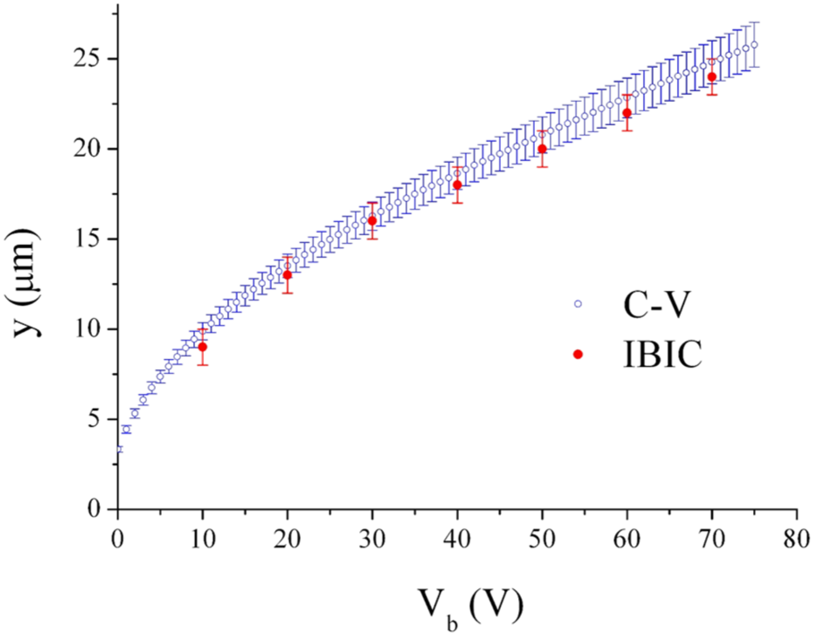
$L_p = (4{,}9 \pm 0{,}3)\,\mu m$

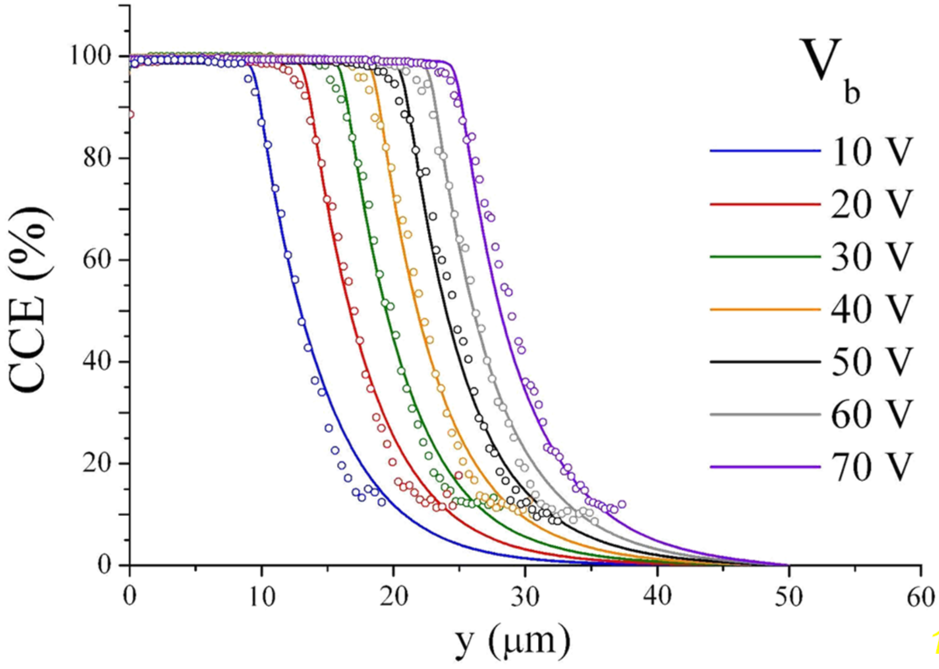
$\tau_p \approx 80\, ns$



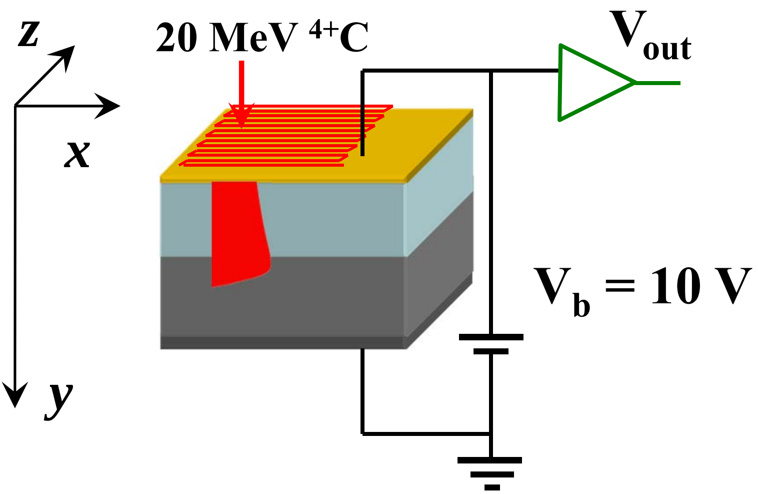


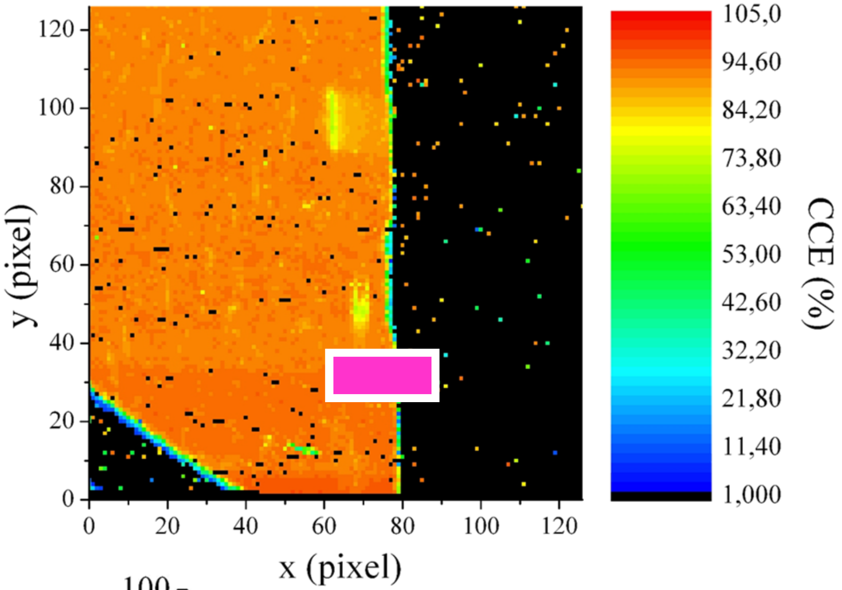


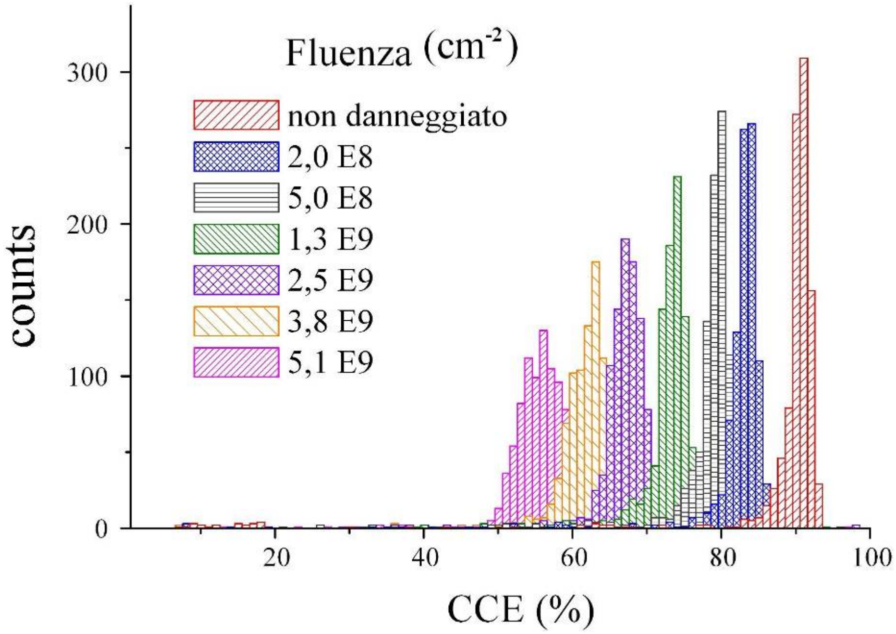


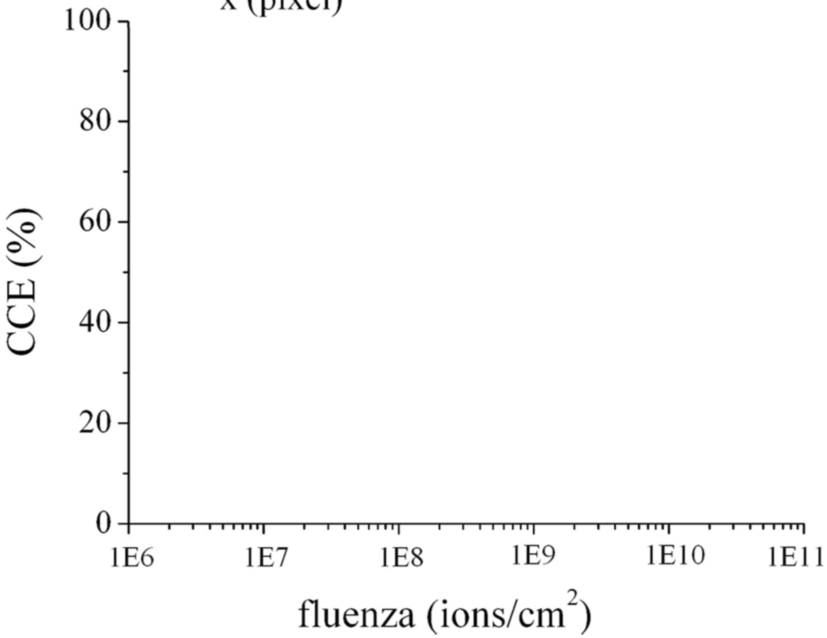


Radiation Damage; Frontal IBIC





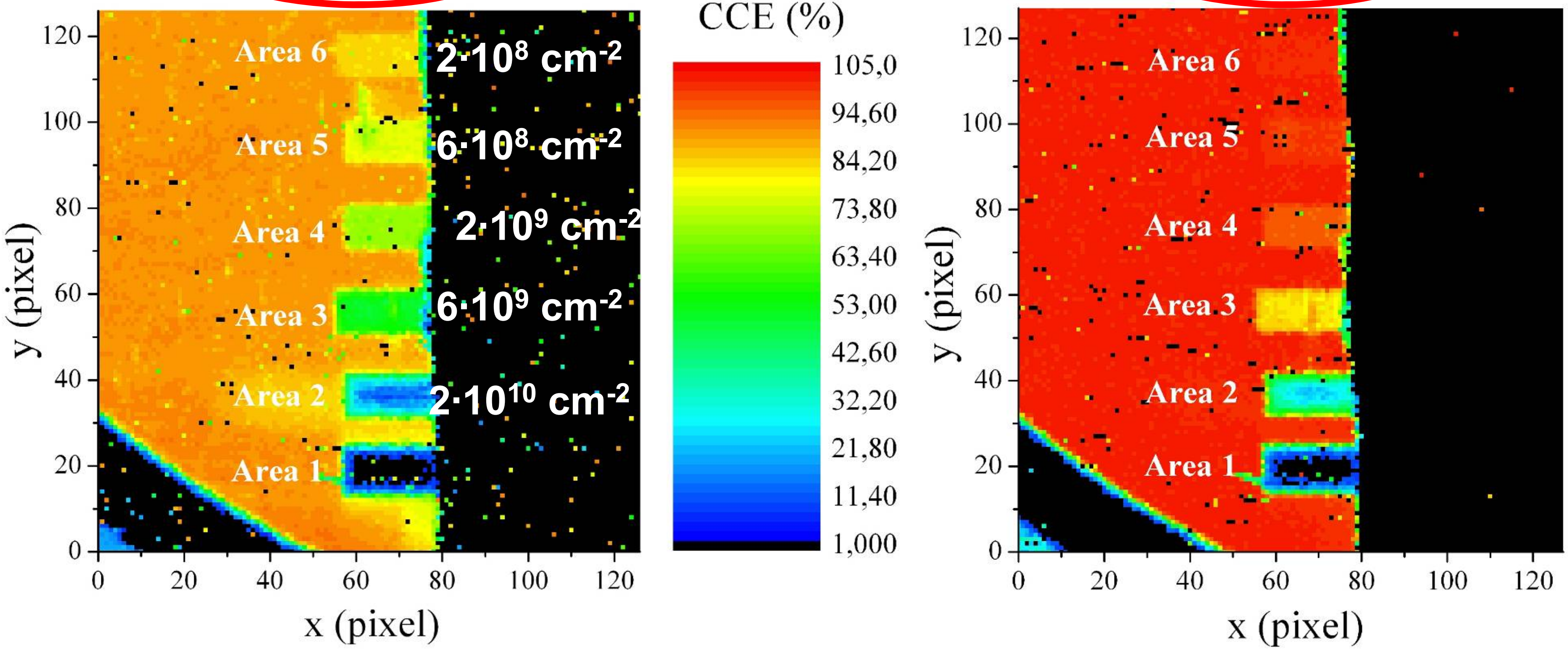




Frontal IBIC

C, 20 MeV
V_b = 10 V

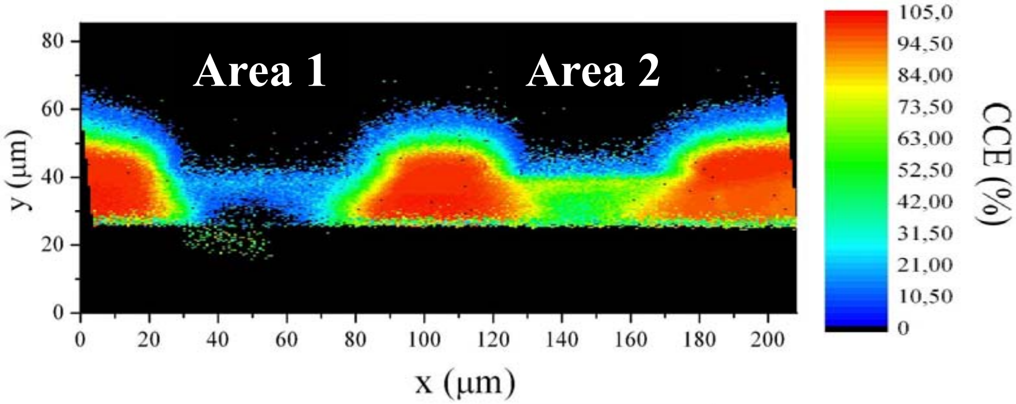
C, 20 MeV
V_b = 50 V

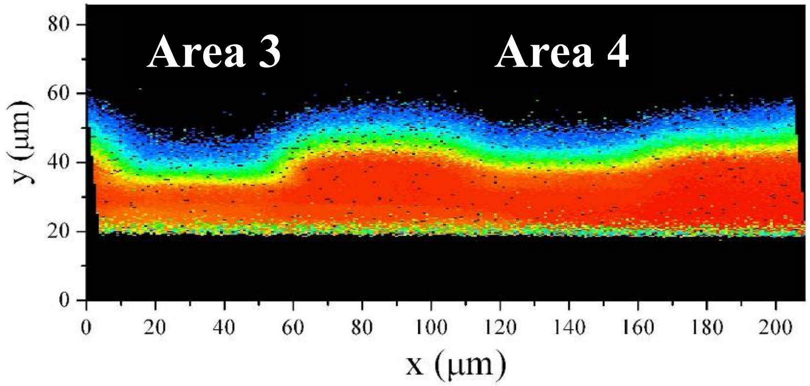


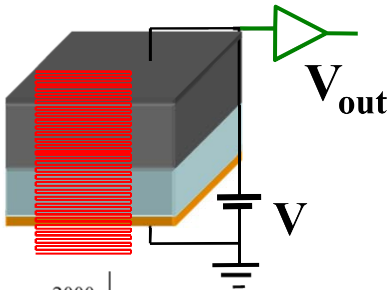
The CCE depends on the ion fluence and on the applied bias voltage

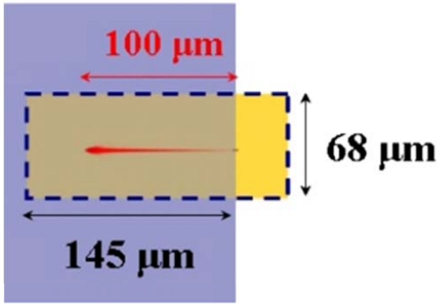
Lateral IBIC

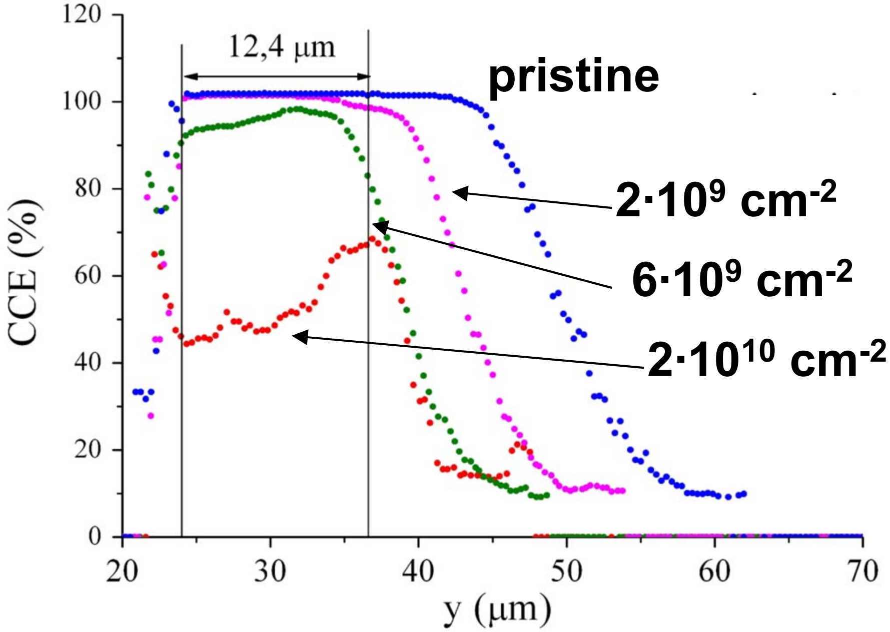
$V_b = 50$ V

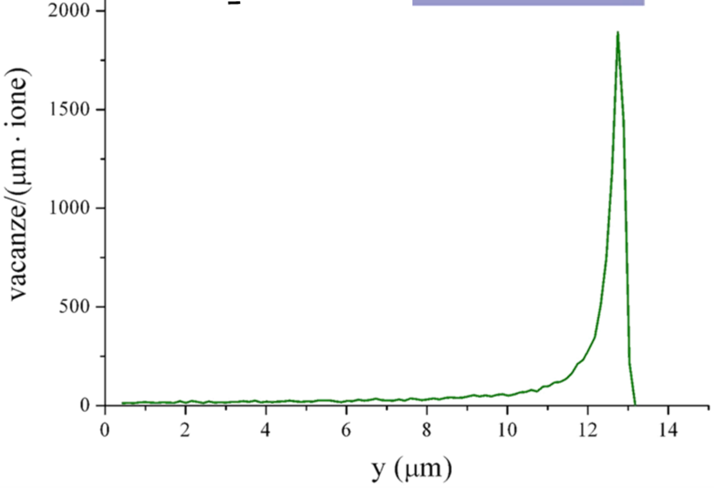




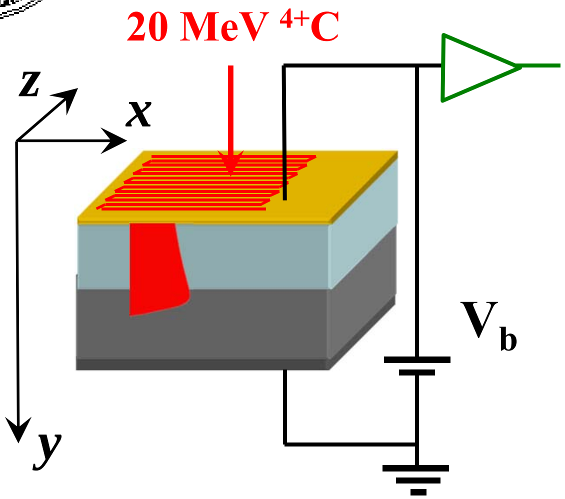




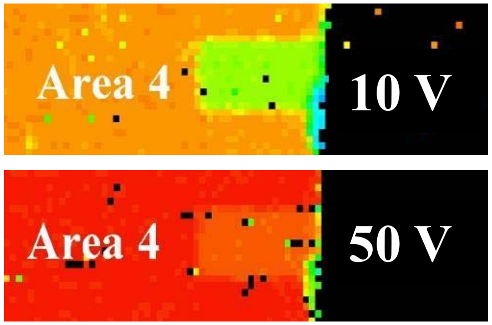


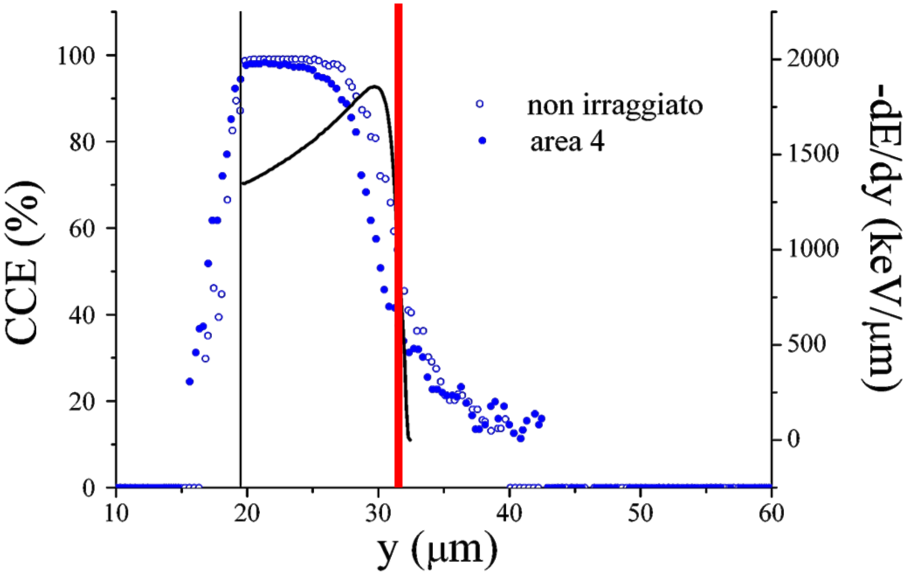


Frontal IBIC

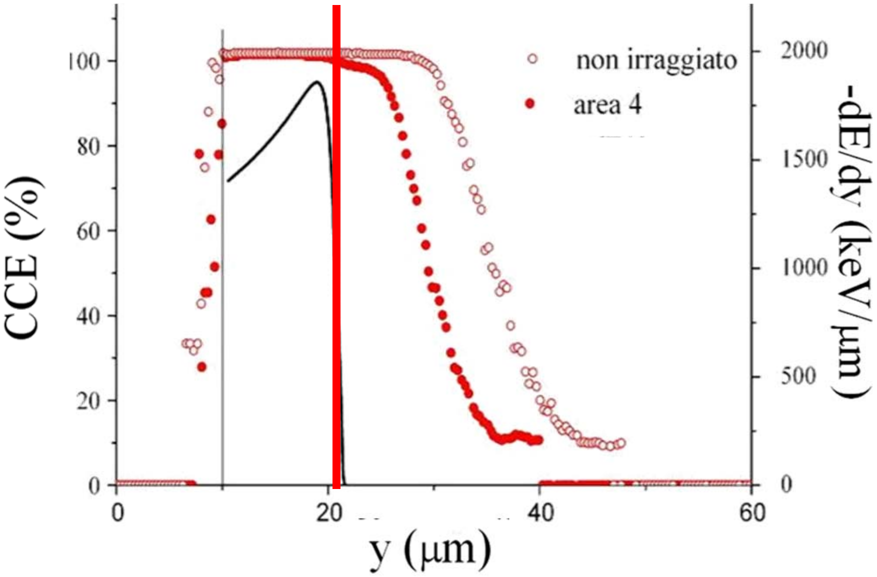


Area 4
($\Phi = 2 \cdot 10^{9}$ cm^{-2})





10 V

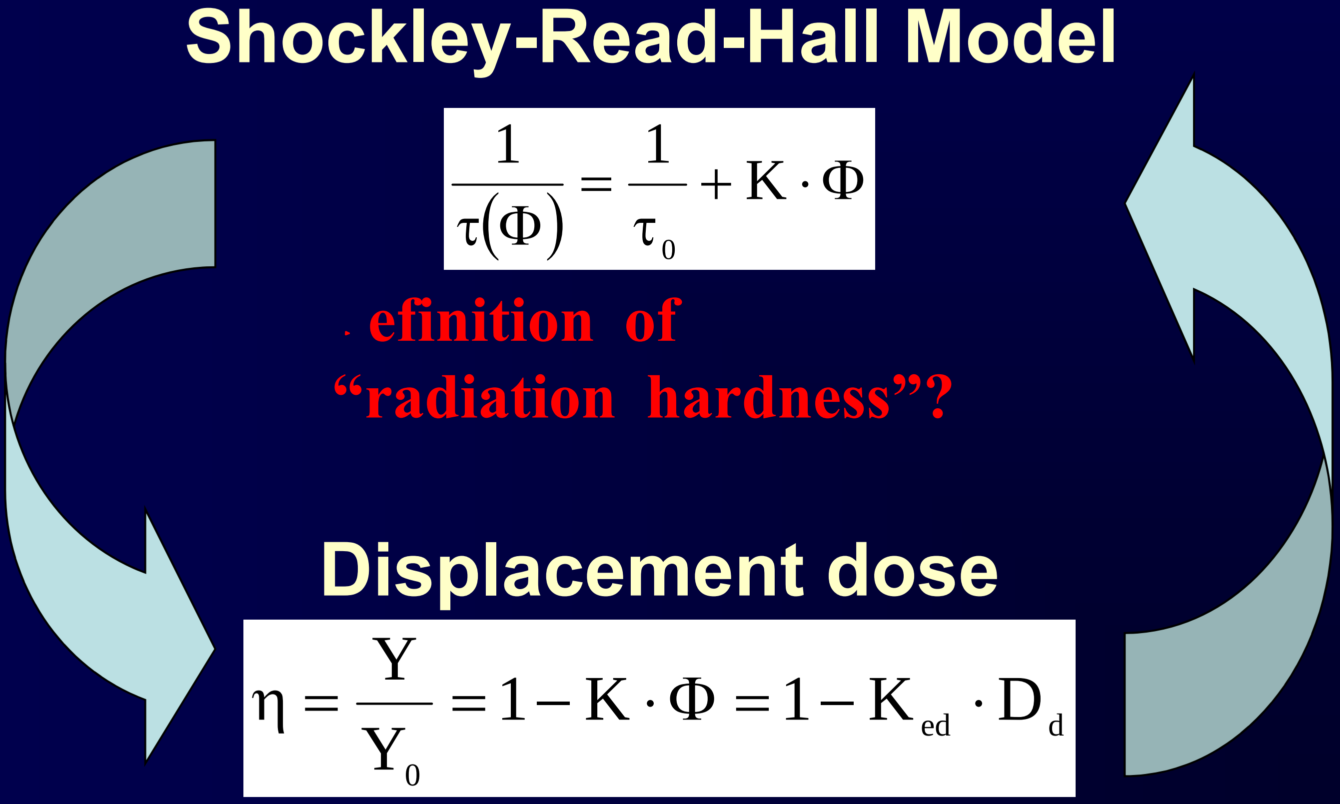


50 V

Lateral IBIC

The performance degradation depends on

- Ion mass and energy
- Polarization state
- Free carrier generation profile (ion probe)



Shockley-Read-Hall Model

$$\frac{1}{\tau(\Phi)} = \frac{1}{\tau_0} + K \cdot \Phi$$

Definition of
“radiation hardness”?

Displacement dose

$$\eta = \frac{Y}{Y_0} = 1 - K \cdot \Phi = 1 - K_{ed} \cdot D_d$$

IAEA Coordinate Research Programme (CRP) F11016 (2011-2015)
“Utilization of ion accelerators for studying and modeling of radiation induced defects in semiconductors and insulators”

\uđer ošković –

Trieste
15.08.2012

Joint ICTP-IAEA Workshop on Physics of Radiation Effect and its Simulation for Non-Metallic Condensed Matter

IAEA Coordinate Research Programme (CRP) F11016 (2011-2015)

“Utilization of ion accelerators for studying and modeling of radiation induced defects in semiconductors and insulators”

Overall Objective:

Use of ion accelerators for improved understanding of how radiation induced defects influence the electronic properties of semiconductor/insulator materials, leading to better understanding of how they degrade or improve the performances of devices in extreme and harsh radiation environments.

Specific Research Objective:

Deeper theoretical knowledge and experimental data on defects created by light and heavy ions; in terms of their type, density and effect on fundamental electronic properties of semiconductors and insulators.

Expected Research Outputs:

Definition of an experimental protocol to determine the key parameters for the characterization of the effects of radiation damage on semiconductor materials and devices.

Refined theoretical models for defect generation and for modelling their effect on electronic properties.

IAEA Coordinate Research Programme (CRP) F11016 (2011-2015)
“Utilization of ion accelerators for studying and modeling of radiation induced defects in semiconductors and insulators”

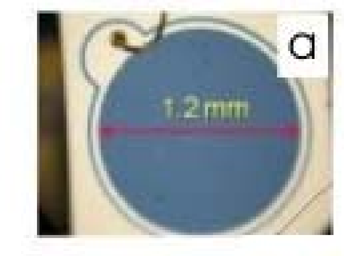
Expected Research Outputs:

Definition of an experimental protocol to determine the key parameters for the characterization of the effects of radiation damage on semiconductor materials and devices.

Refined theoretical models for defect generation and for modelling their effect on electronic properties.

Definition of an experimental protocol

Hamamatsu
S5821 p-i-n diode

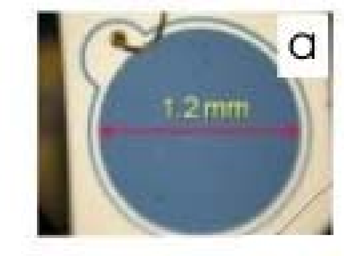


Experimental protocol

- ✓Commercial p-i-n diodes

Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)

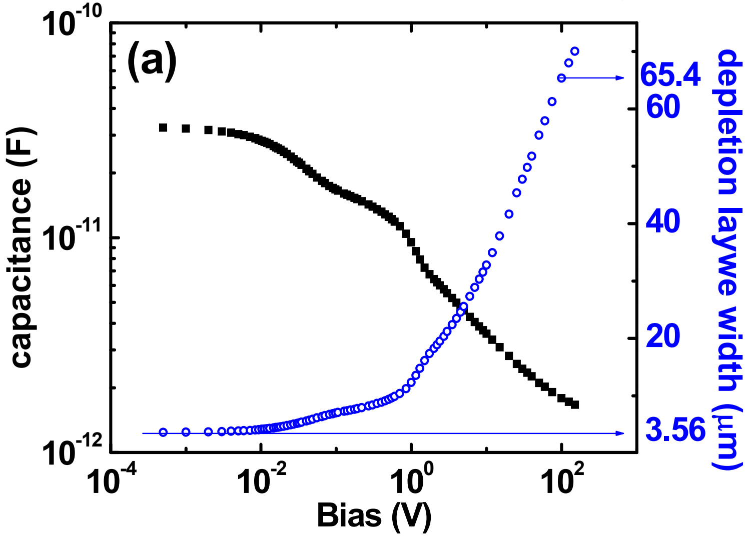
Hamamatsu
S5821 p-i-n diode



Experimental protocol

- ✓Commercial p-i-n diodes
- ✓Electrical characterization

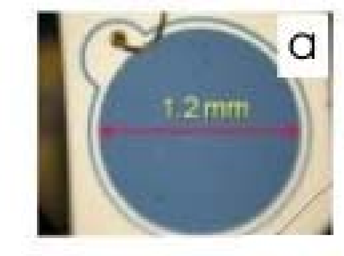
C-V characteristics
Depletion width-voltage

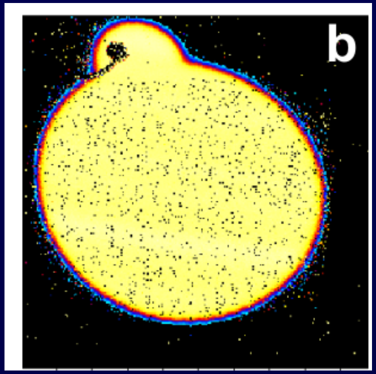


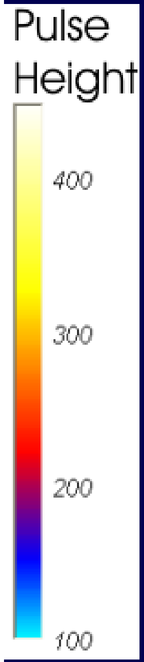
Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)

IBIC map on a pristine diode probed with a scanning 1.4 MeV He microbeam;

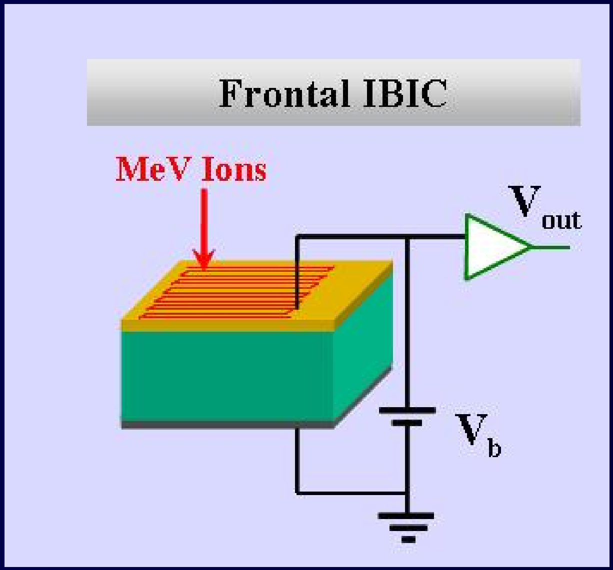
Hamamatsu S5821 p-i-n diode







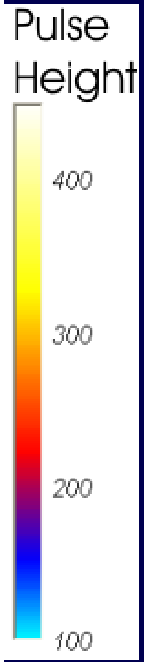
Uniform CCE map



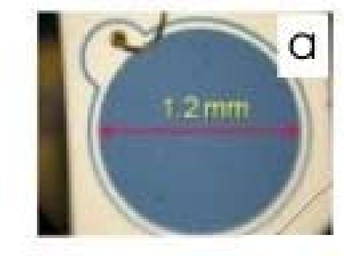
Experimental protocol

- ✓Commercial p-i-n diodes
- ✓Electrical characterization
- ✓IBIC map on pristine sample

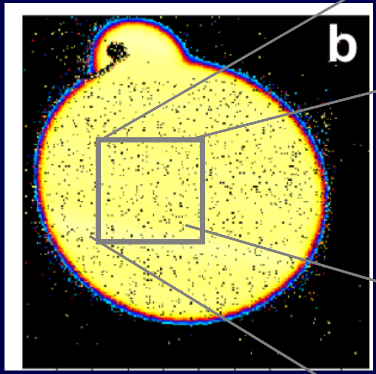
Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)



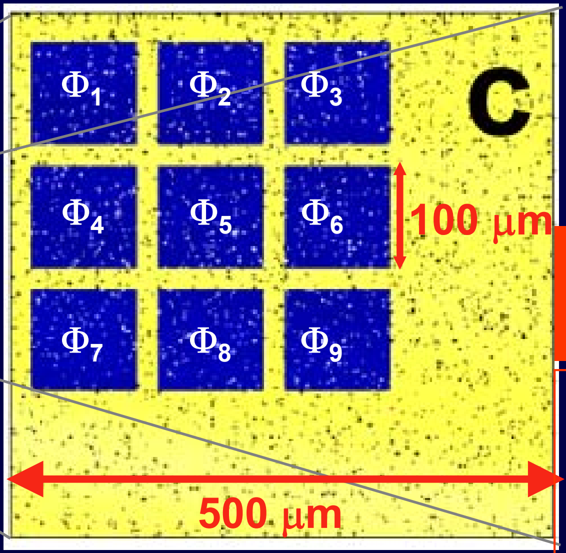
Hamamatsu S5821 p-i-n diode



IBIC map on a pristine diode probed with a scanning 1.4 MeV He microbeam;



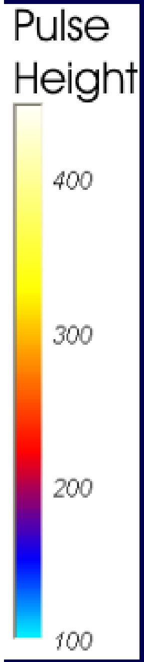
ZOOM in view of the selected area for focused ion beam irradiation at different fluences Φ



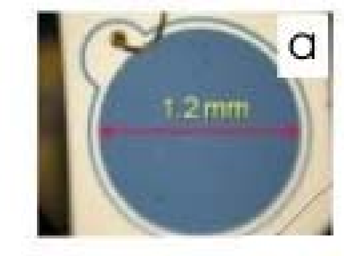
Experimental protocol

- ✓Commercial p-i-n diodes
- ✓Electrical characterization
- ✓IBIC map on pristine sample
- ✓Irradiation of 9 regions at different fluences

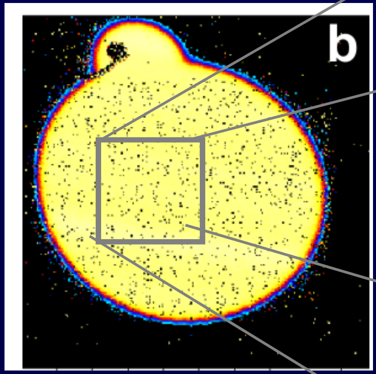
Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)



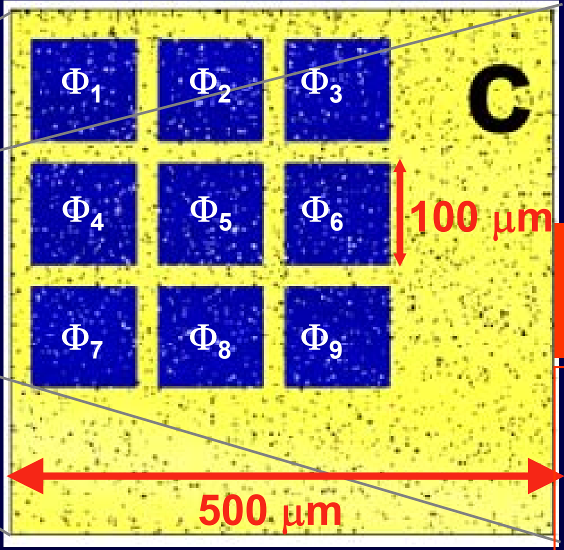
Hamamatsu S5821 p-i-n diode



IBIC map on a pristine diode probed with a scanning 1.4 MeV He microbeam;

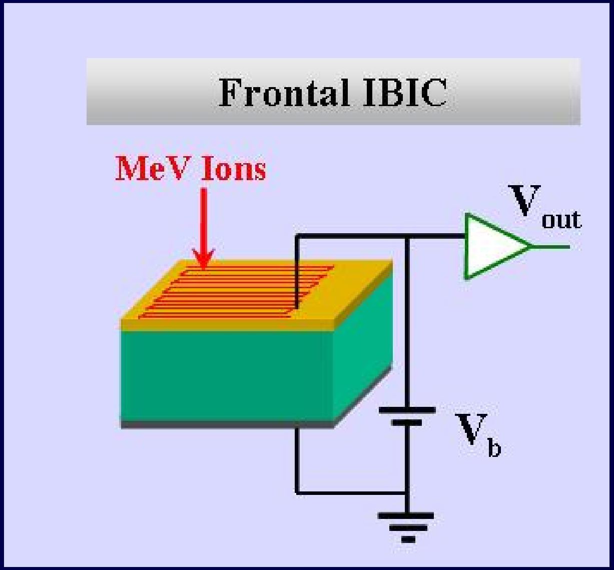


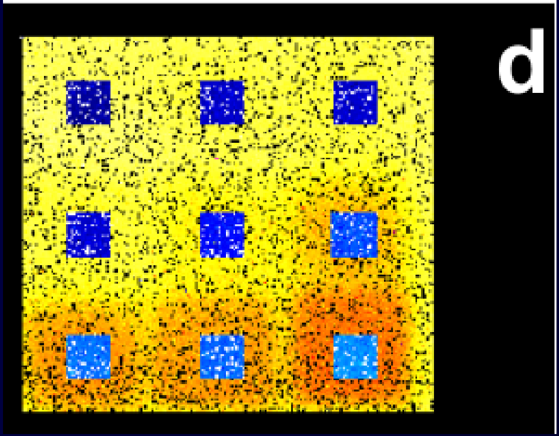
ZOOM in view of the selected area for focused ion beam irradiation at different fluences Φ



Experimental protocol

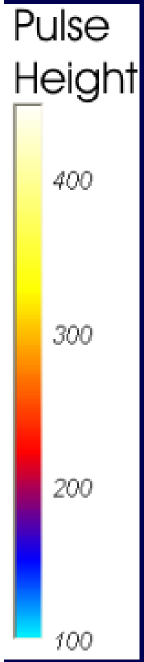
- ✓Commercial p-i-n diodes
- ✓Electrical characterization
- ✓IBIC map on pristine sample
- ✓Irradiatoin of 9 regions at different fluences
- ✓IBIC map of irradiated regions



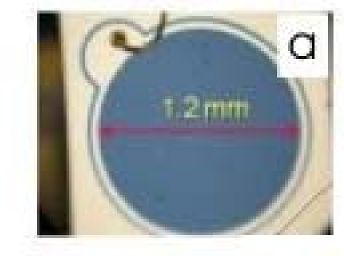


a measured 2D distribution of the IBIC signal amplitude after irradiation

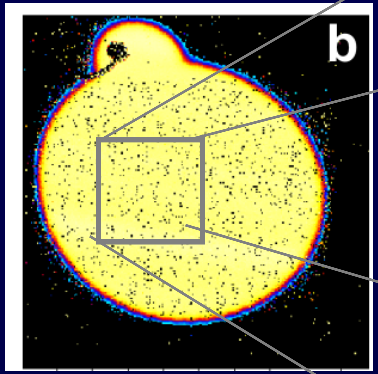
Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)



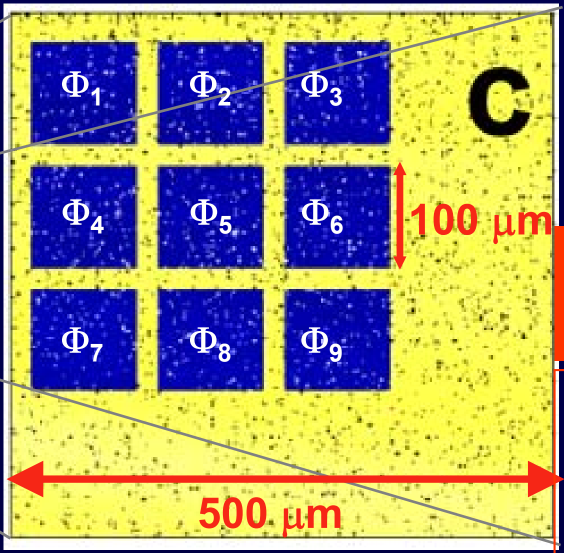
Hamamatsu S5821 p-i-n diode



IBIC map on a pristine diode probed with a scanning 1.4 MeV He microbeam;



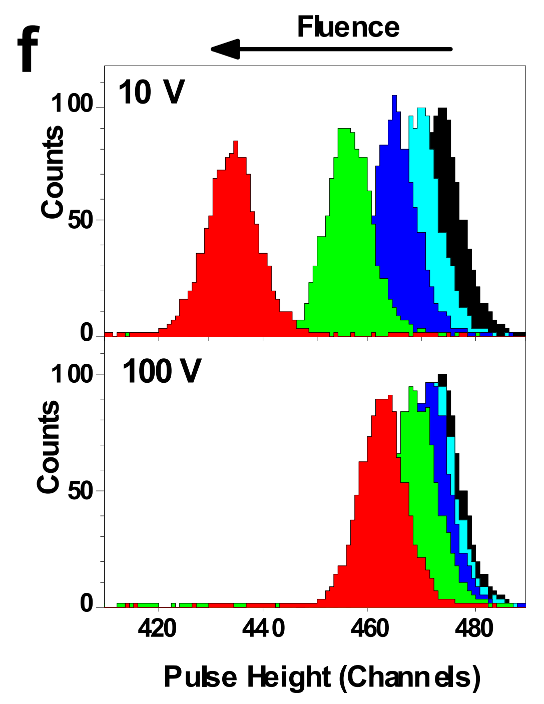
ZOOM in view of the selected area for focused ion beam irradiation at different fluences Φ

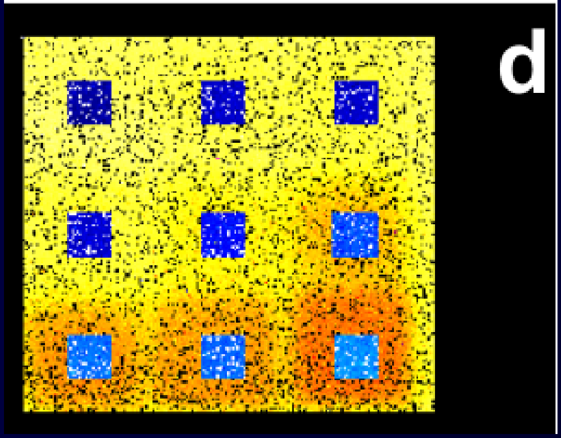


Experimental protocol

- ✓Commercial p-i-n diodes
- ✓Electrical characterization
- ✓IBIC map on pristine sample
- ✓Irradiatin of 9 regions at different fluences
- ✓IBIC map of irradiated regions
- ✓Average pulse height as function of the damage

IBIC spectra (bias voltage = 10 V and 100 V) from the central regions of four of the areas shown in Fig. c

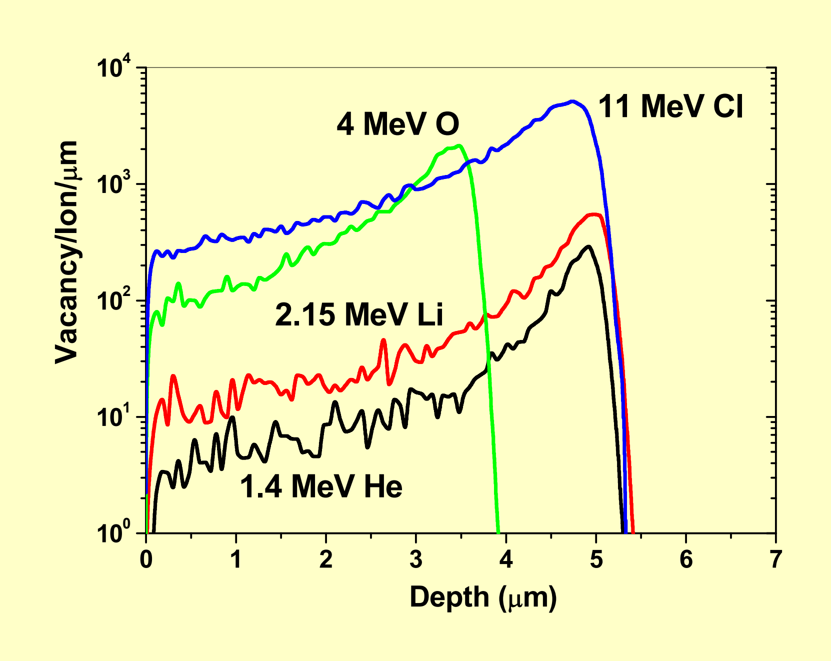




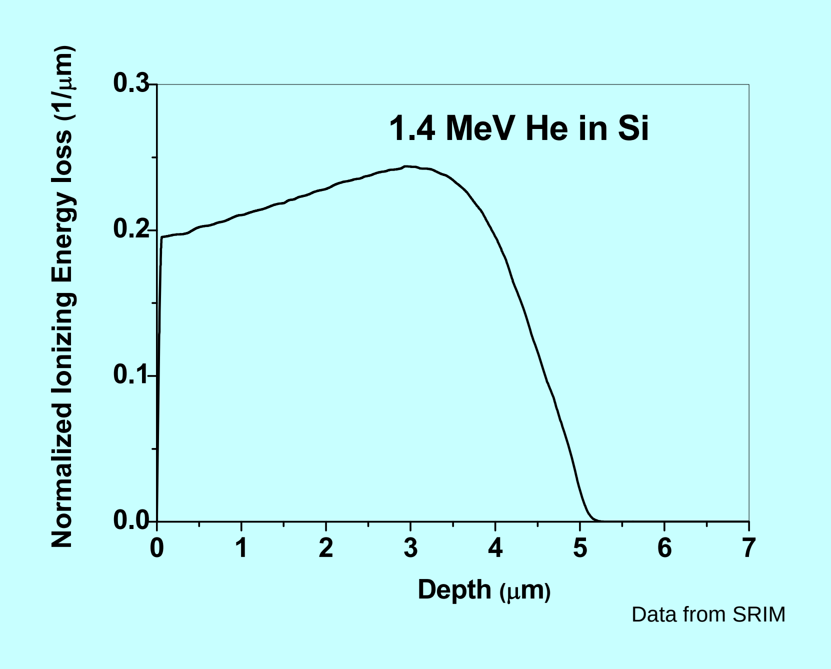
a measured 2D distribution of the IBIC signal amplitude after irradiation

Z. Pastuovic et al., IEEE Trans on Nucl. Sc. 56 (2009) 2457; APL (98) 092101 (2011)

Damaging Ions

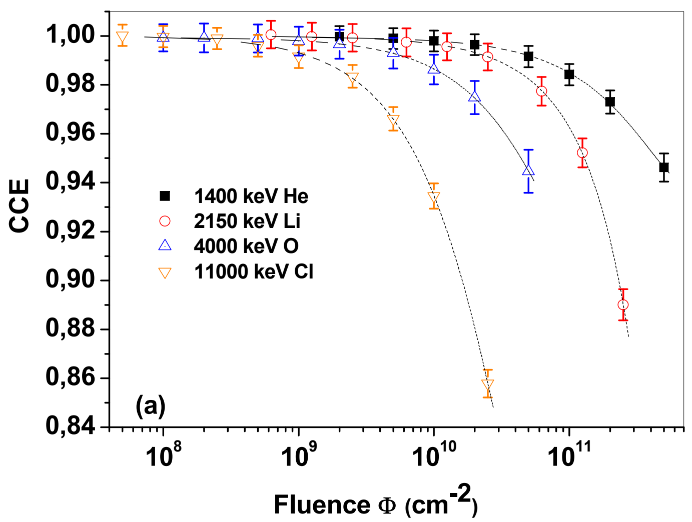


Ion Probe

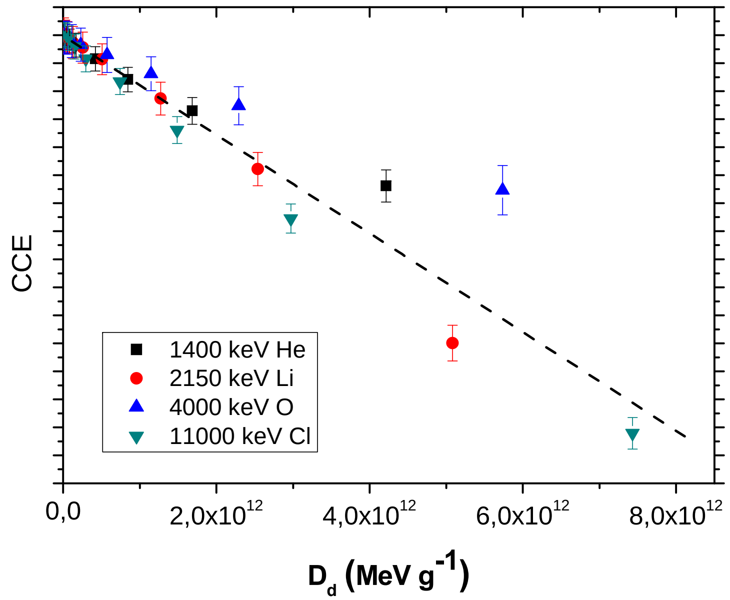


Data from SRIM

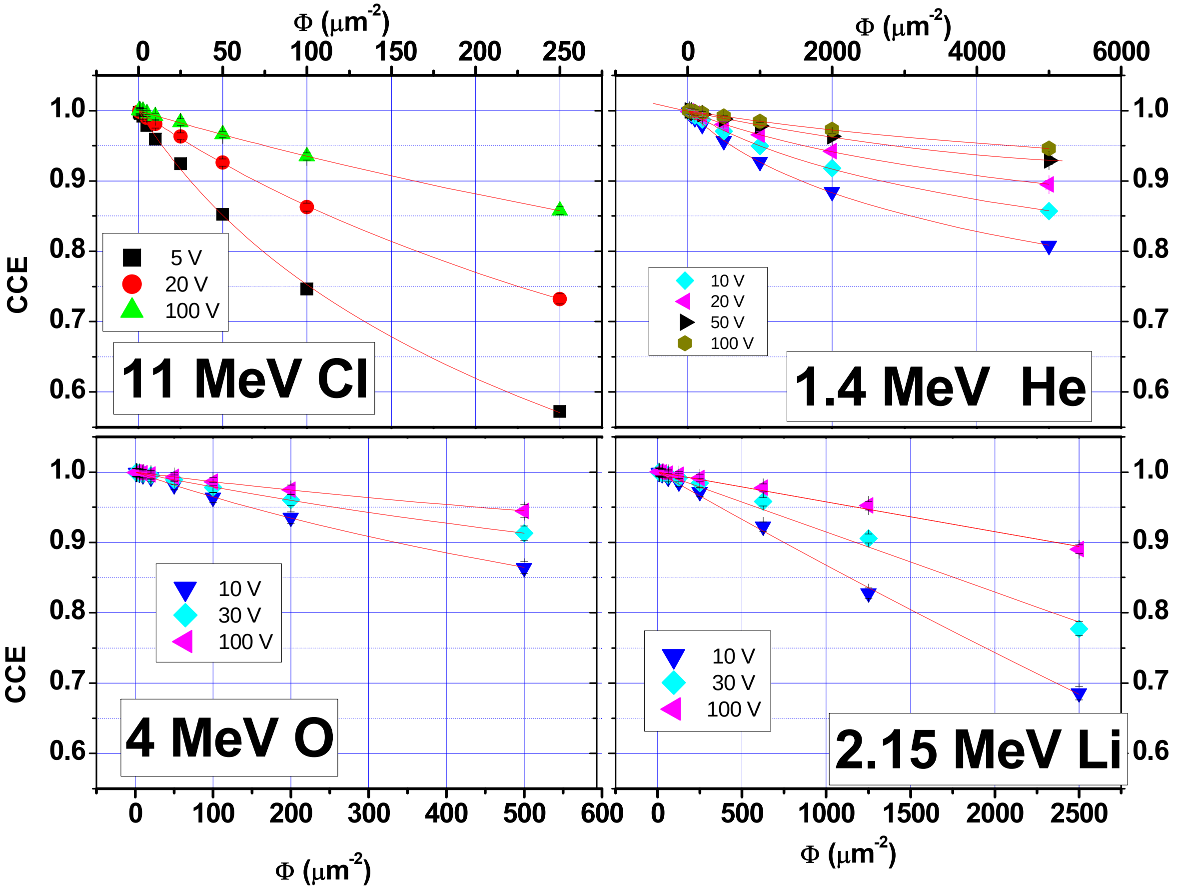
V_{bias}=100 V
Fully depleted device



CCE behavior in regions damaged with different ions vs. ion fluence (Φ); the dashed lines are parabolic fits as guides for eyes.



The same data points shown in Fig. 4 for plotted against the adjusted damage dose D_d.



Measured CCE values for 1400 keV He ion detection in selected areas of biased Hamamatsu S5821 diodes irradiated with different fluences 1.4 MeV He, 2.15 MeV Li , 4.0 MeV O and 11 MeV Cl ions. The dashed lines are parabolic fits as guides for eyes.

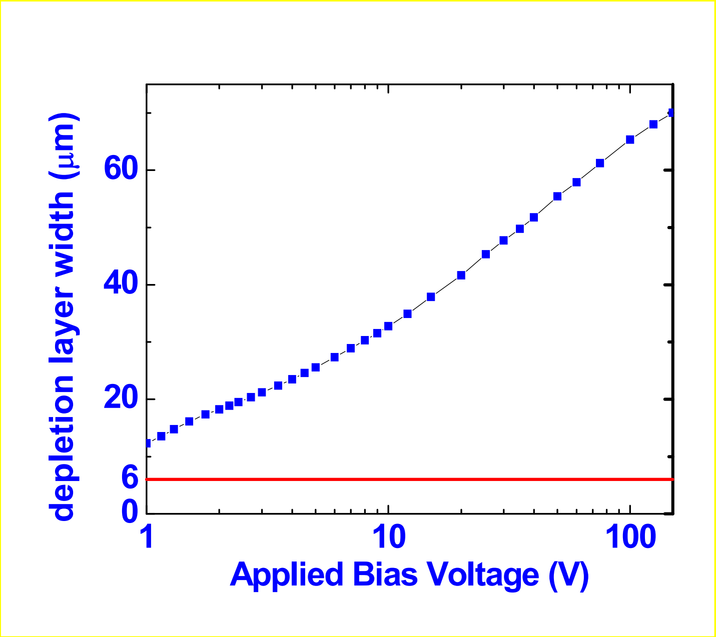
IAEA Coordinate Research Programme (CRP) F11016 (2011-2015)
“Utilization of ion accelerators for studying and modeling of radiation induced defects in semiconductors and insulators”

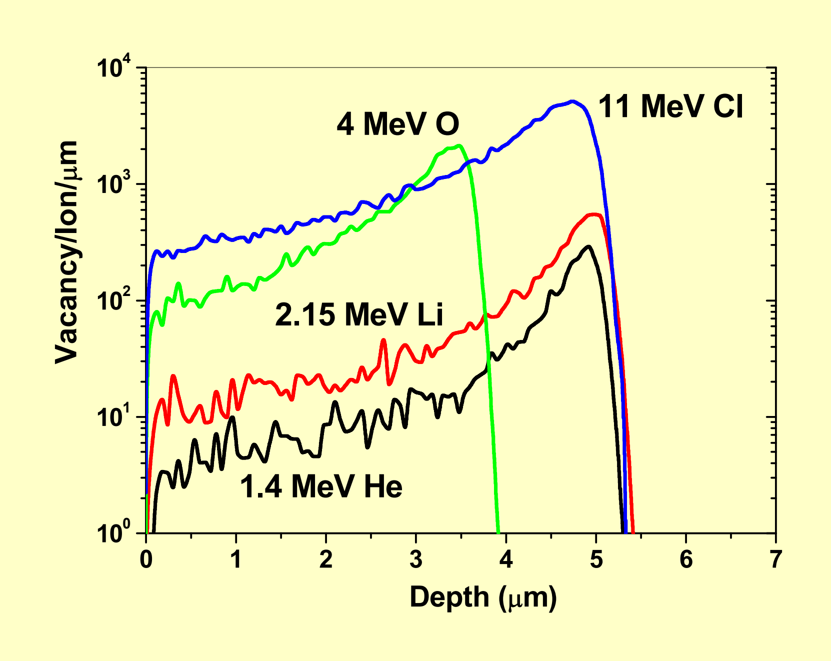
Expected Research Outputs:

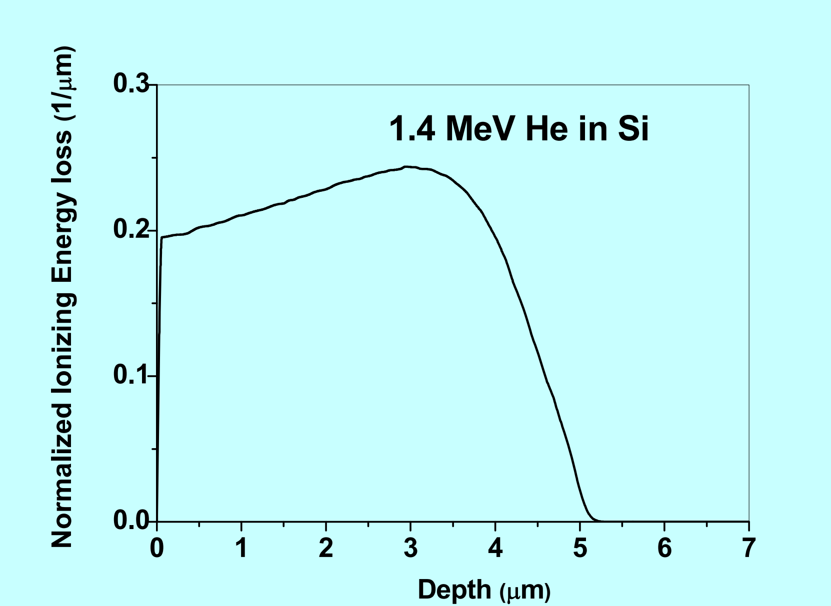
Definition of an experimental protocol to determine the key parameters for the characterization of the effects of radiation damage on semiconductor materials and devices.

Refined theoretical models for defect generation and for modelling their effect on electronic properties.

Fully depleted device





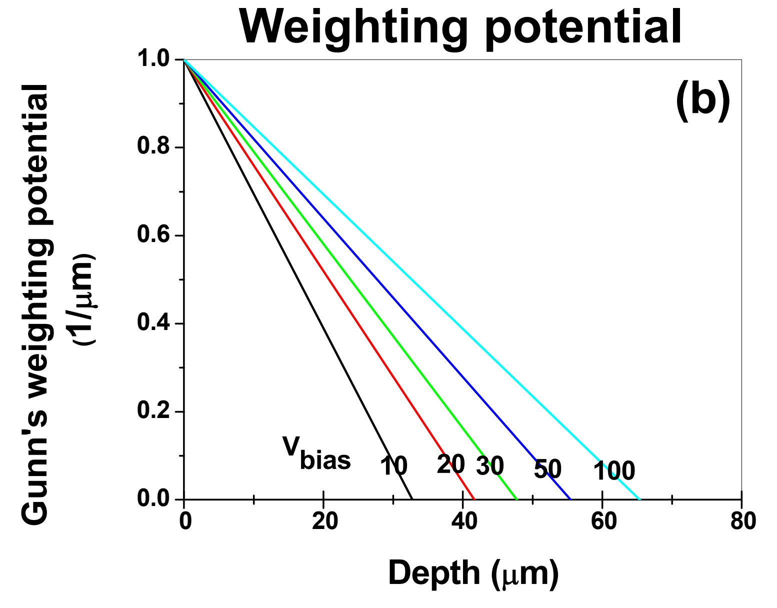


Data from SRIM

Fully depleted

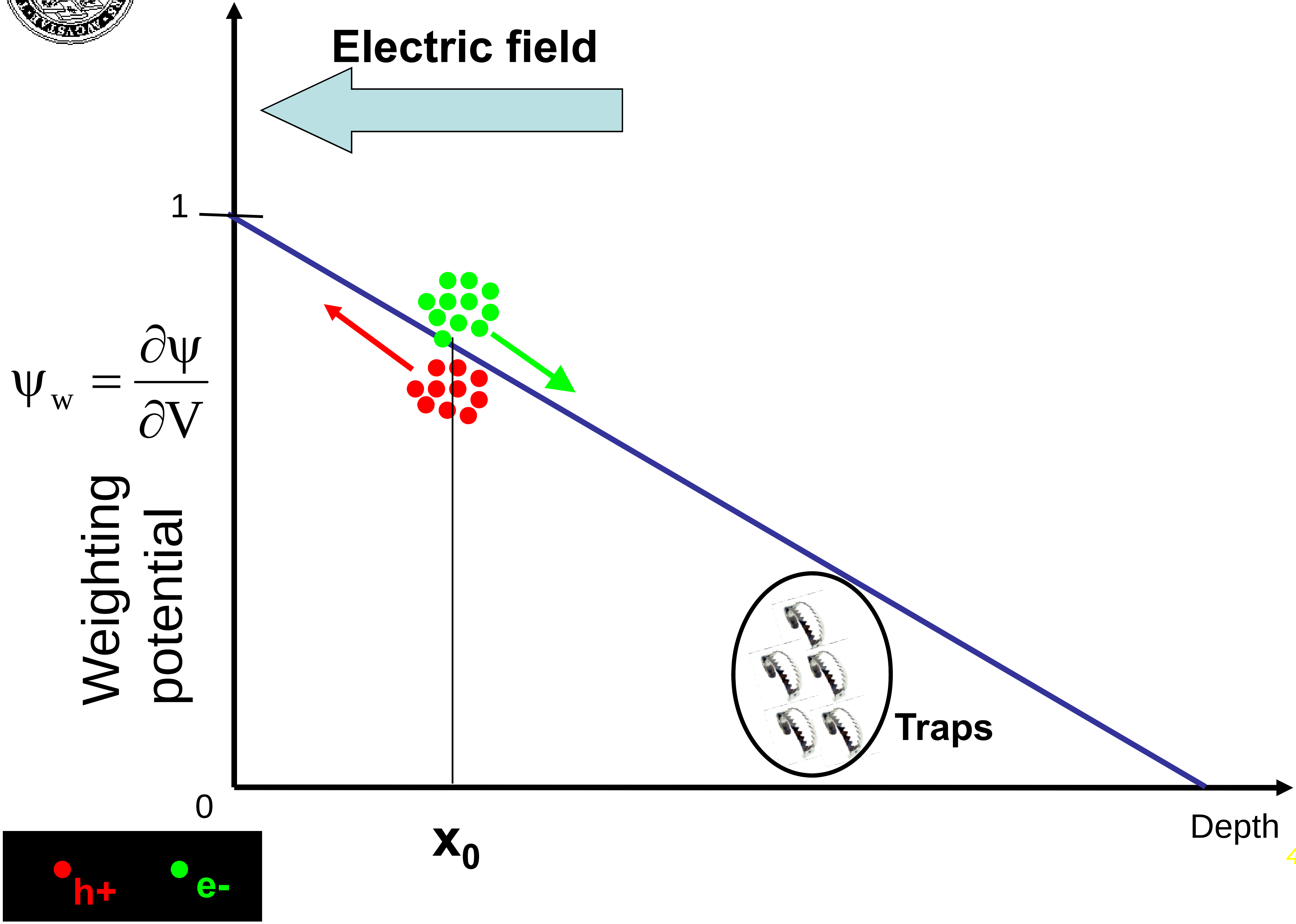
Ramo Theorem
(no diffusion)

$$\left(\varphi = \frac{\partial\psi}{\partial V_{\text{bias}}}\right)$$



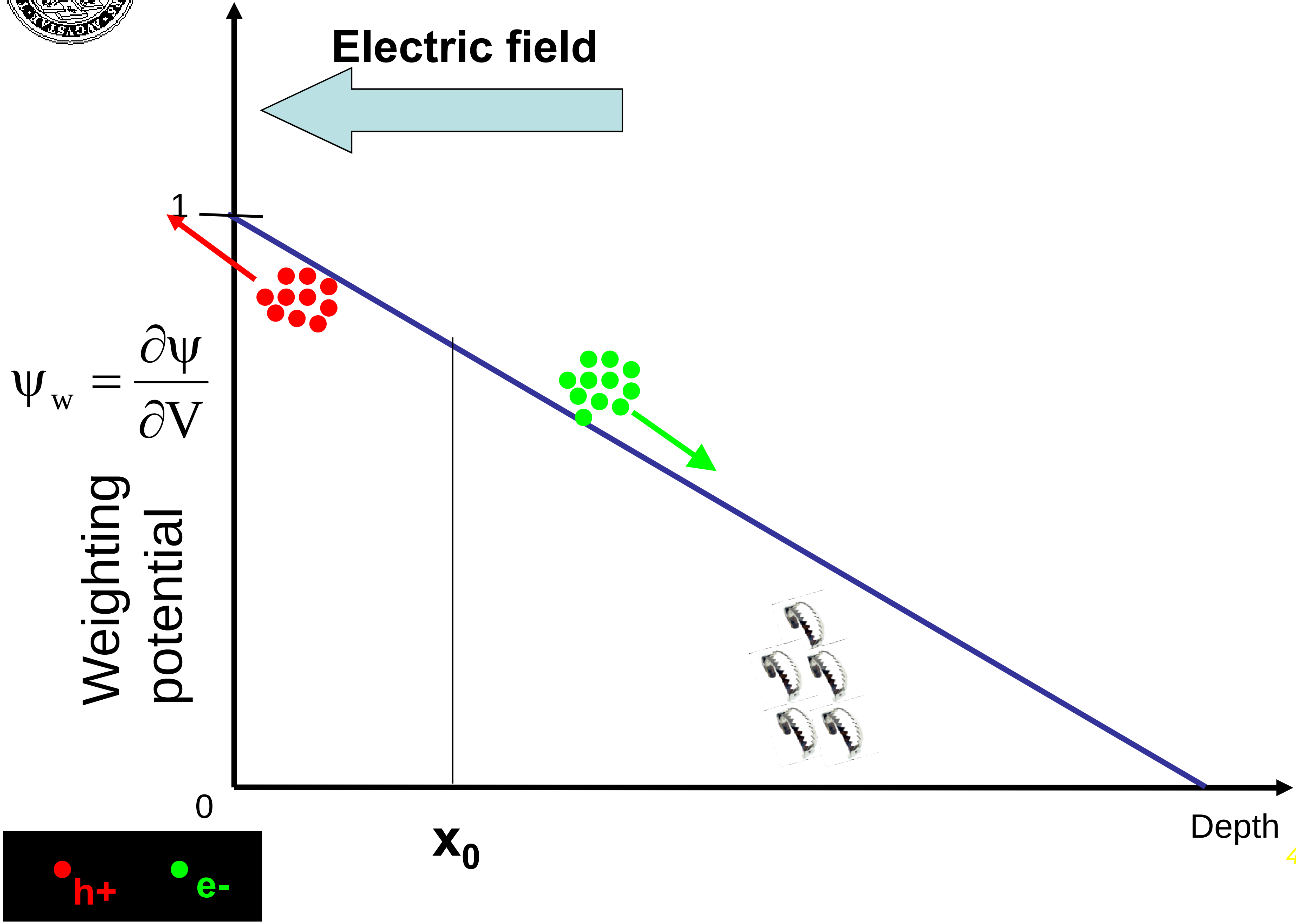
$$\varphi = \frac{\partial\psi}{\partial V} = \begin{cases} \left(1 - \frac{x}{w}\right) & \text{for } x < w \\ 0 & \text{for } x > w \end{cases}$$

Effects of localized recombination centres



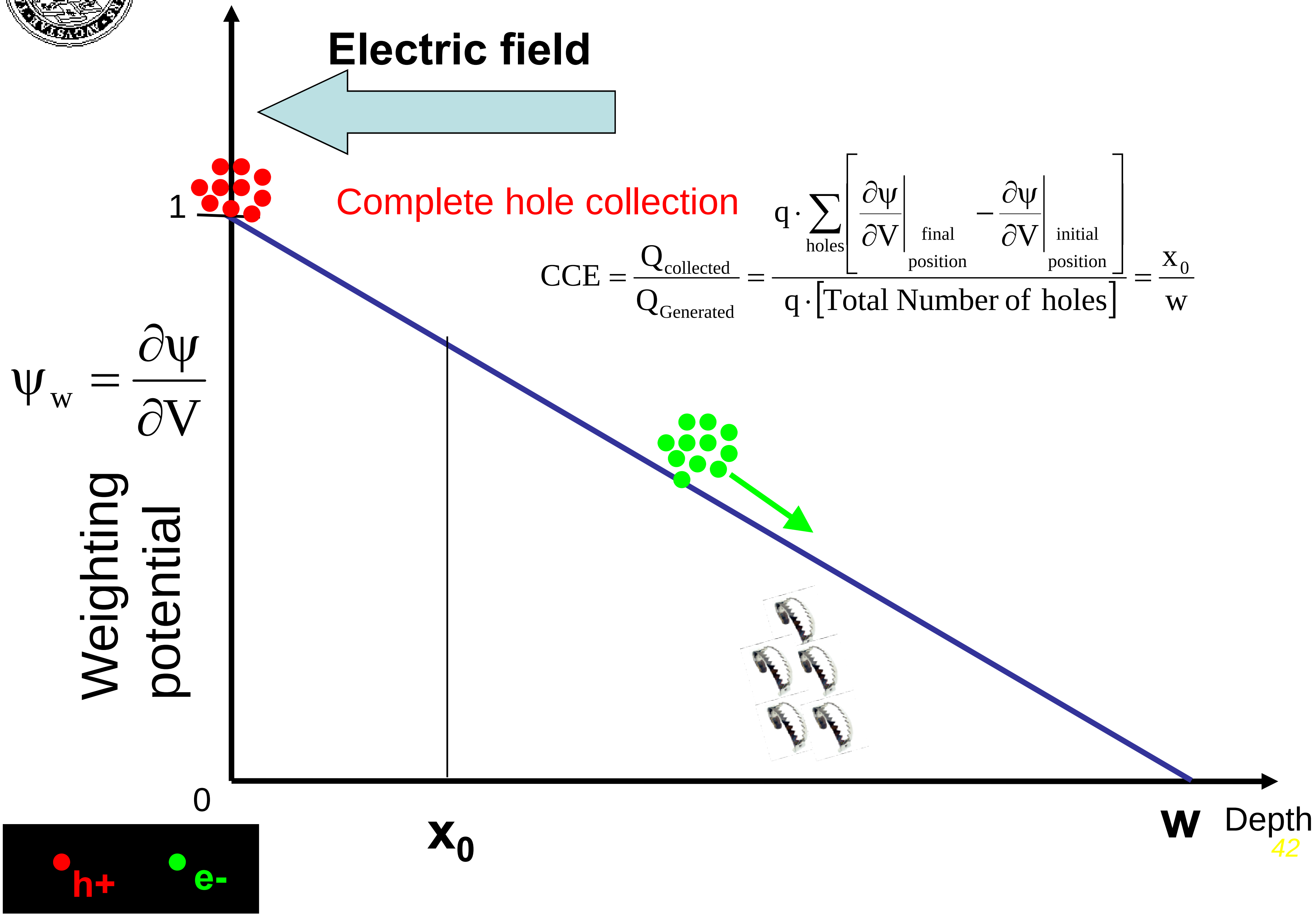
$$\psi_w = \frac{\partial\psi}{\partial V}$$

Effects of localized recombination centres



$$\psi_w = \frac{\partial \psi}{\partial V}$$

Effects of localized recombination centres

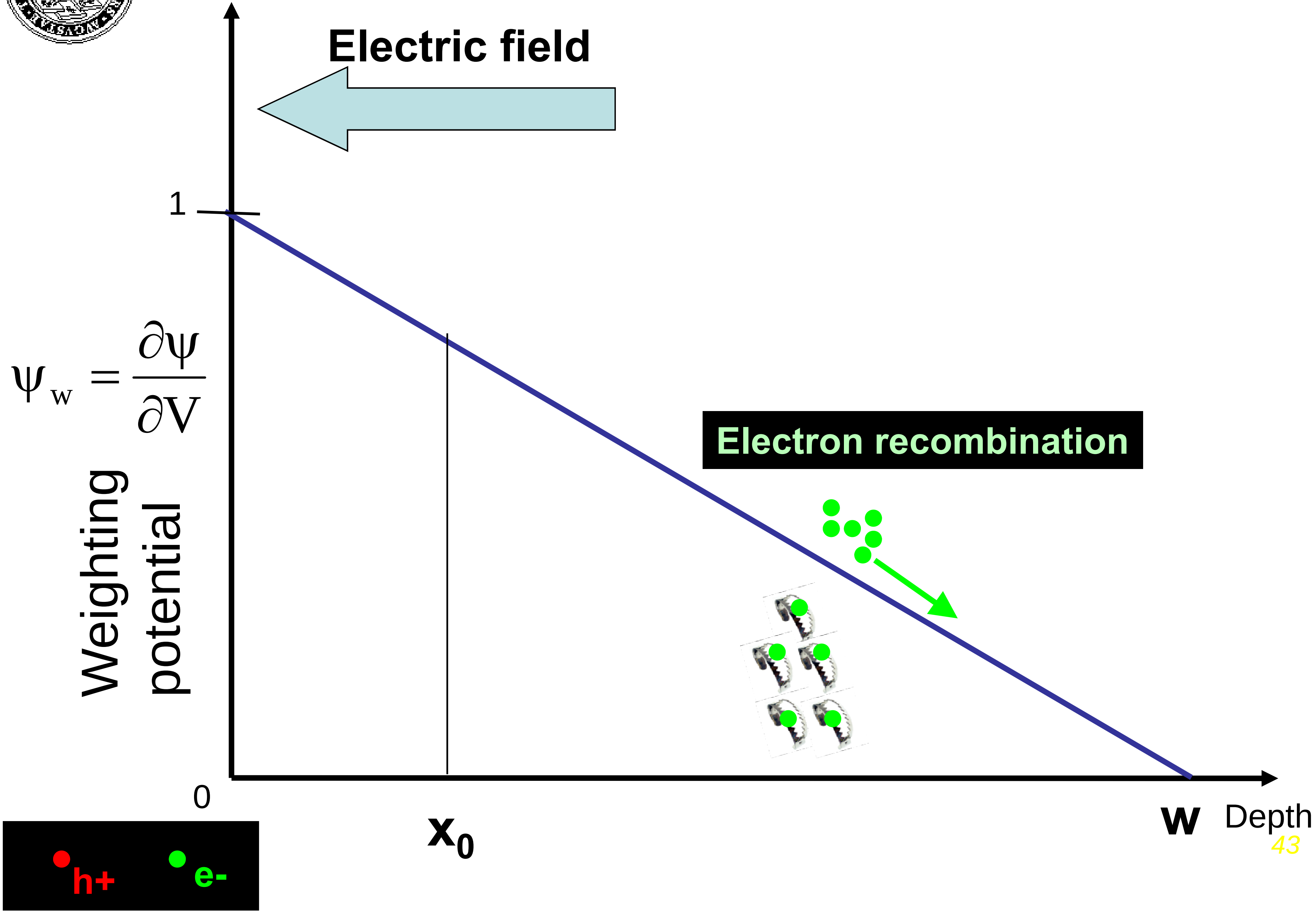


Complete hole collection

$$CCE = \frac{Q_{collected}}{Q_{Generated}} = \frac{q\cdot\sum_{holes}\left[\left.\frac{\partial\psi}{\partial V}\right|_{\substack{final\\position}} - \left.\frac{\partial\psi}{\partial V}\right|_{\substack{initial\\position}}\right]}{q\cdot\left[\text{Total Number of holes}\right]} = \frac{x_0}{w}$$

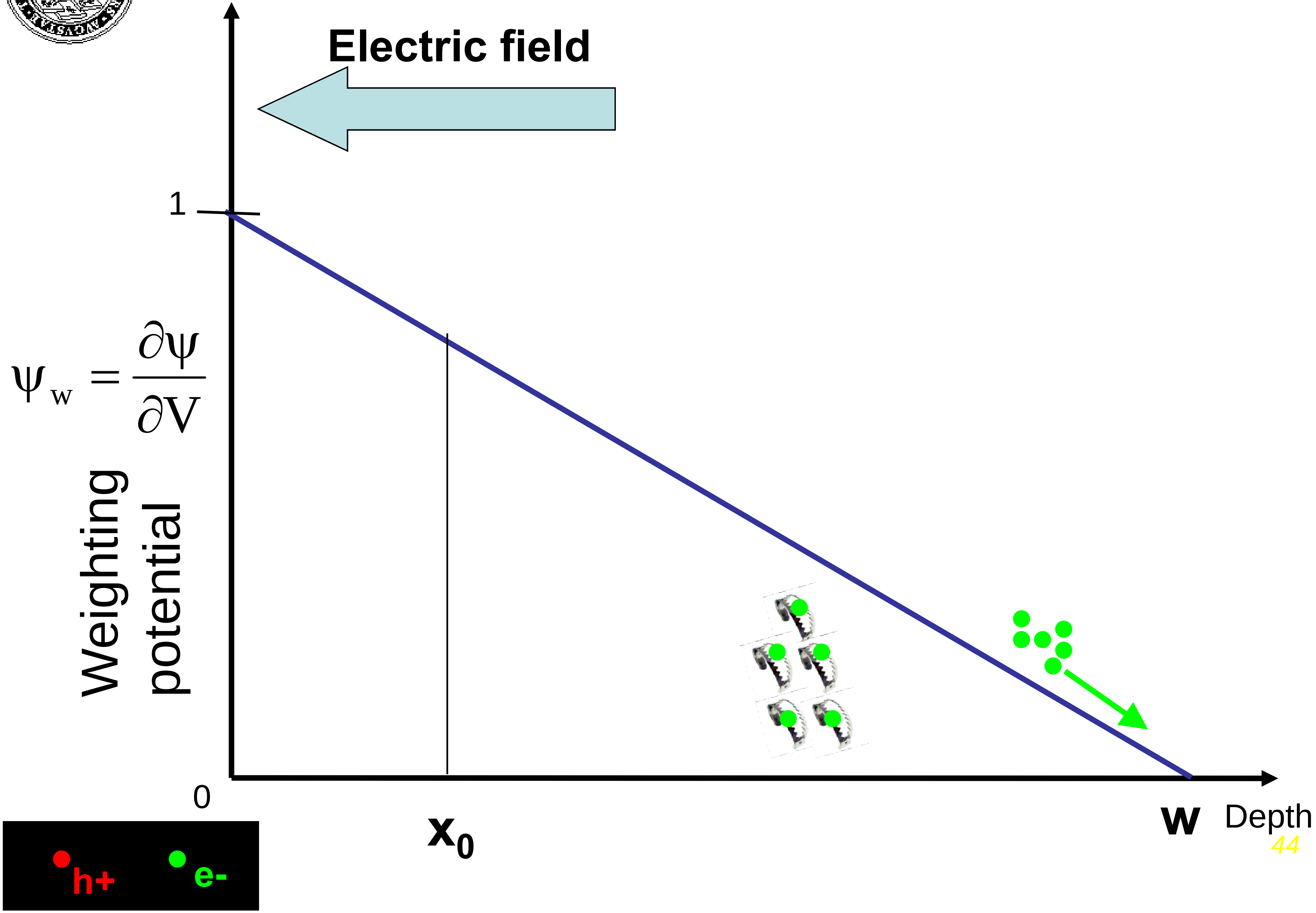
$$\psi_w = \frac{\partial\psi}{\partial V}$$

Effects of localized recombination centres



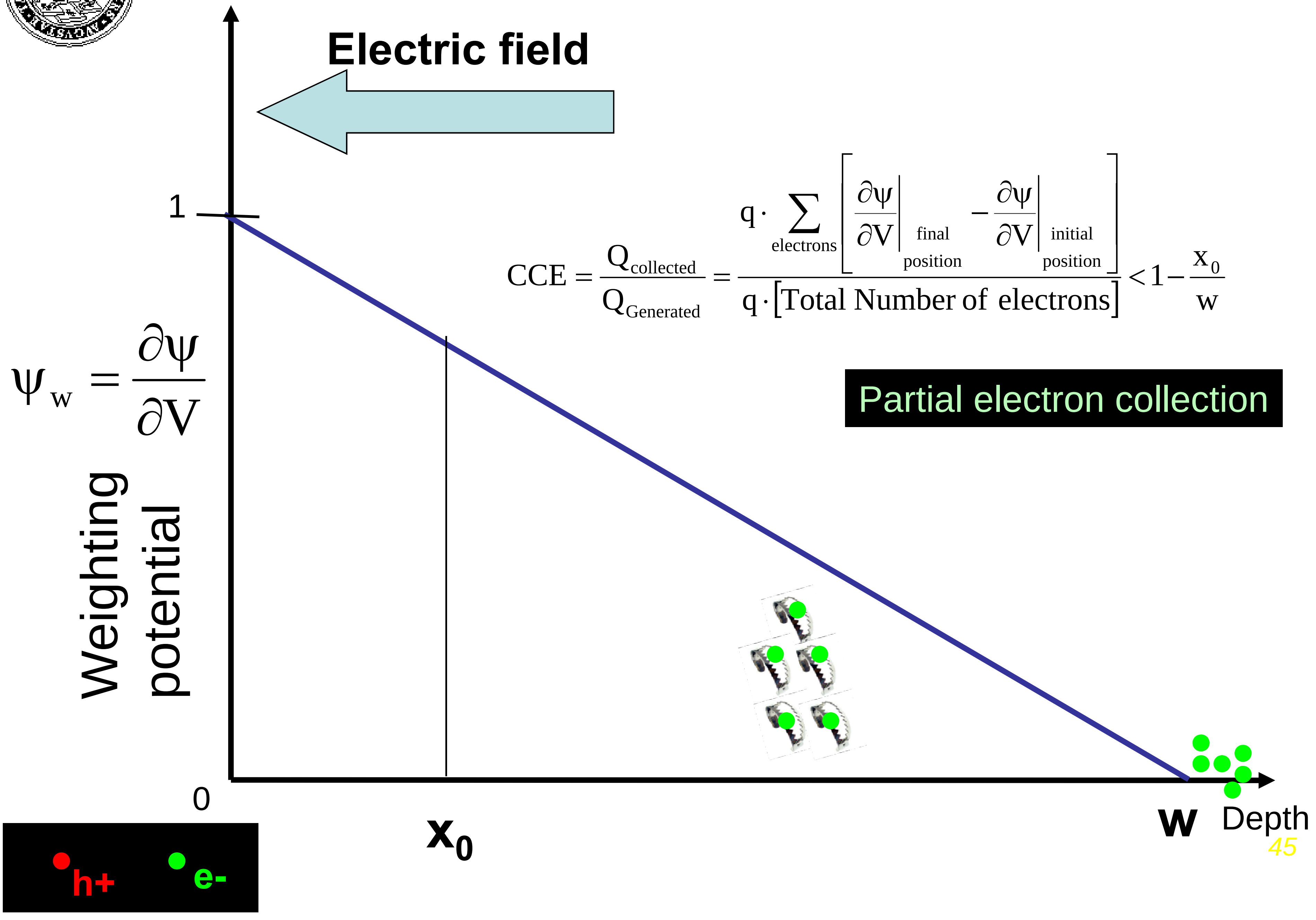
$$\psi_w = \frac{\partial \psi}{\partial V}$$

Effects of localized recombination centres



$$\psi_w = \frac{\partial \psi}{\partial V}$$

Effects of localized recombination centres



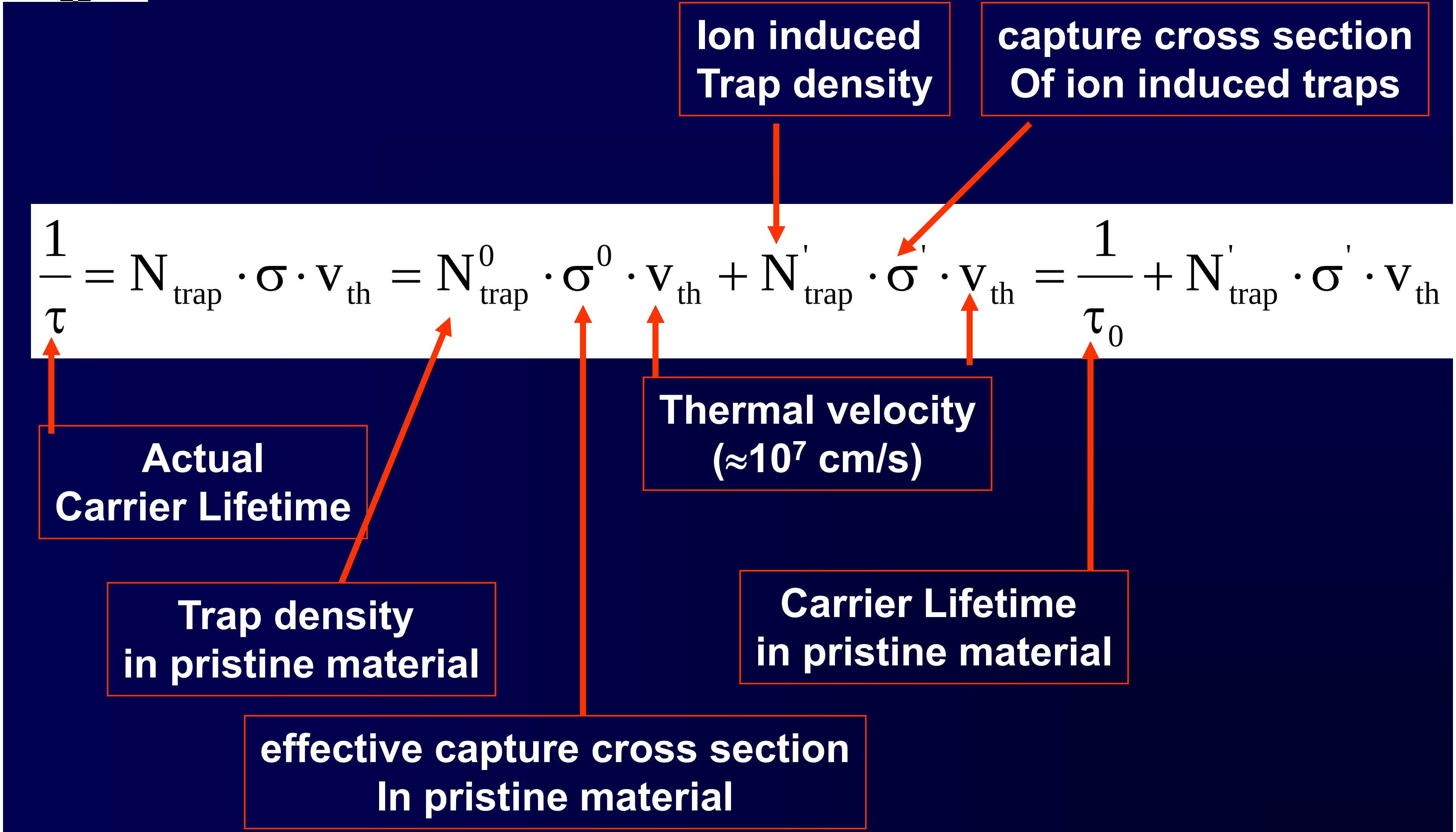
Electric field

$$\psi_w = \frac{\partial \psi}{\partial V}$$

$$\mathrm{CCE} = \frac{Q_{\mathrm{collected}}}{Q_{\mathrm{Generated}}} = \frac{q \cdot \sum_{\mathrm{electrons}} \left[\left.\frac{\partial \psi}{\partial V}\right|_{\substack{\mathrm{final}\\ \mathrm{position}}} - \left.\frac{\partial \psi}{\partial V}\right|_{\substack{\mathrm{initial}\\ \mathrm{position}}} \right]}{q \cdot [\text{Total Number of electrons}]} < 1 - \frac{x_0}{w}$$

Partial electron collection

Shockley-Read-Hall model



$$\frac{1}{\tau} = N_{trap}\cdot\sigma\cdot v_{th} = N^{0}_{trap}\cdot\sigma^{0}\cdot v_{th} + N^{'}_{trap}\cdot\sigma^{'}\cdot v_{th} = \frac{1}{\tau_0} + N^{'}_{trap}\cdot\sigma^{'}\cdot v_{th}$$

Ion induced
Trap density

capture cross section
Of ion induced traps

Actual
Carrier Lifetime

Thermal velocity
(≈10^7 cm/s)

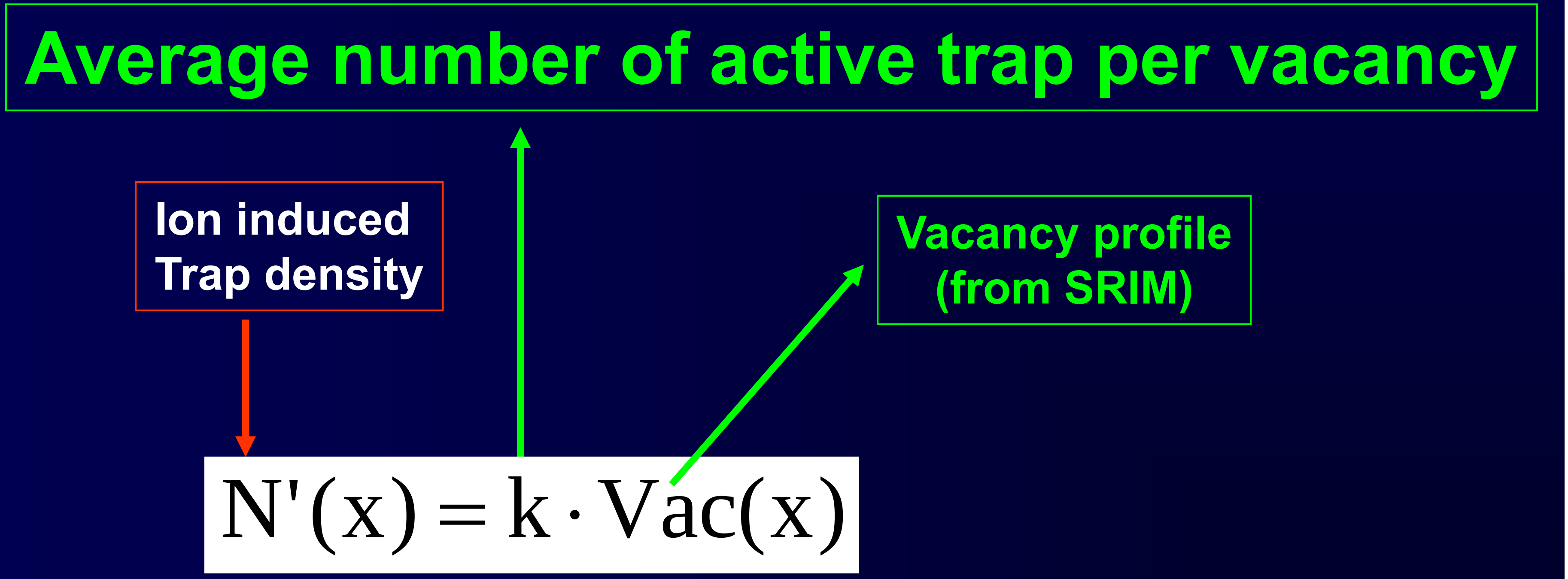
Trap density
in pristine material

Carrier Lifetime
in pristine material

effective capture cross section
In pristine material

Shockley-Read-Hall model

$$\frac{1}{\tau}=\frac{1}{\tau_0}+N'_{\mathrm{trap}}\cdot\sigma'\cdot v_{\mathrm{th}}$$



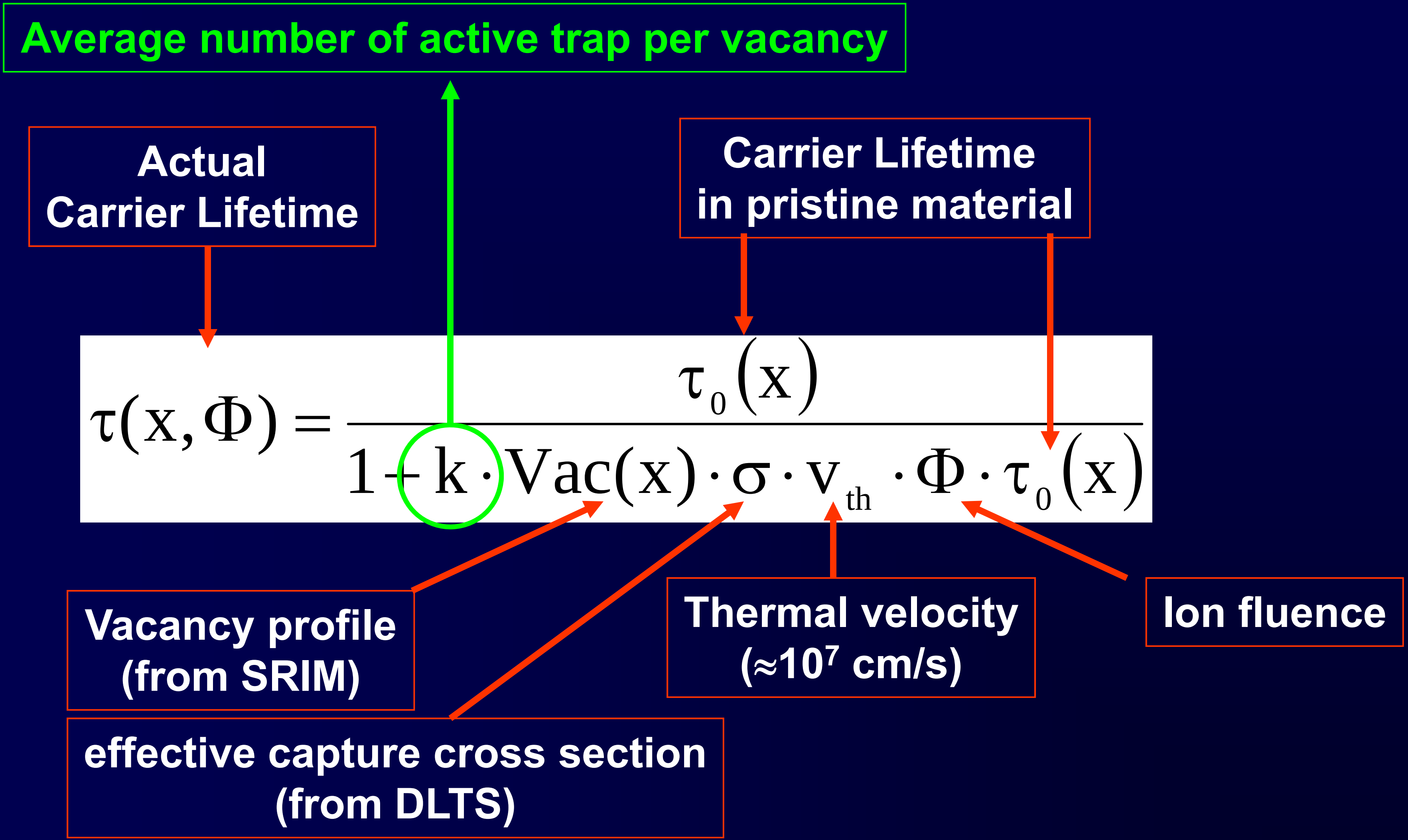
Average number of active trap per vacancy

Ion induced Trap density

Vacancy profile (from SRIM)

$$N'(x)=k\cdot Vac(x)$$

Shockley-Read-Hall model



Average number of active trap per vacancy

Actual Carrier Lifetime

Carrier Lifetime in pristine material

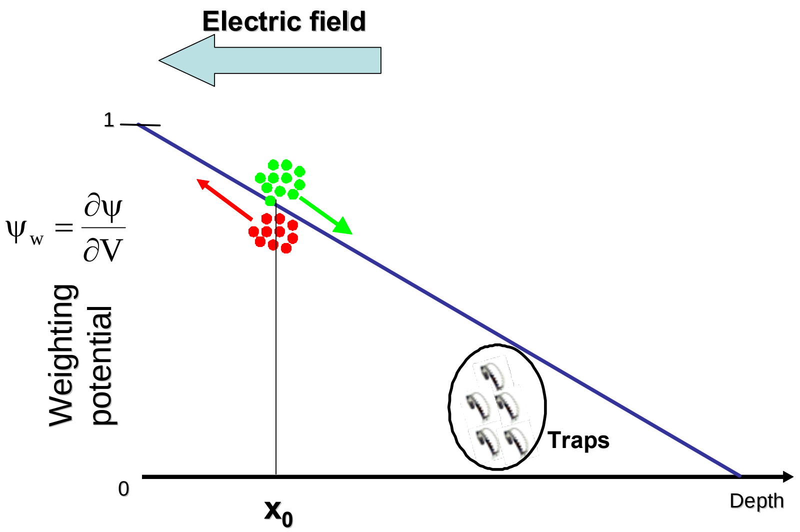
$$\tau(x,\Phi) = \frac{\tau_0(x)}{1 + k \cdot \mathrm{Vac}(x) \cdot \sigma \cdot v_{th} \cdot \Phi \cdot \tau_0(x)}$$

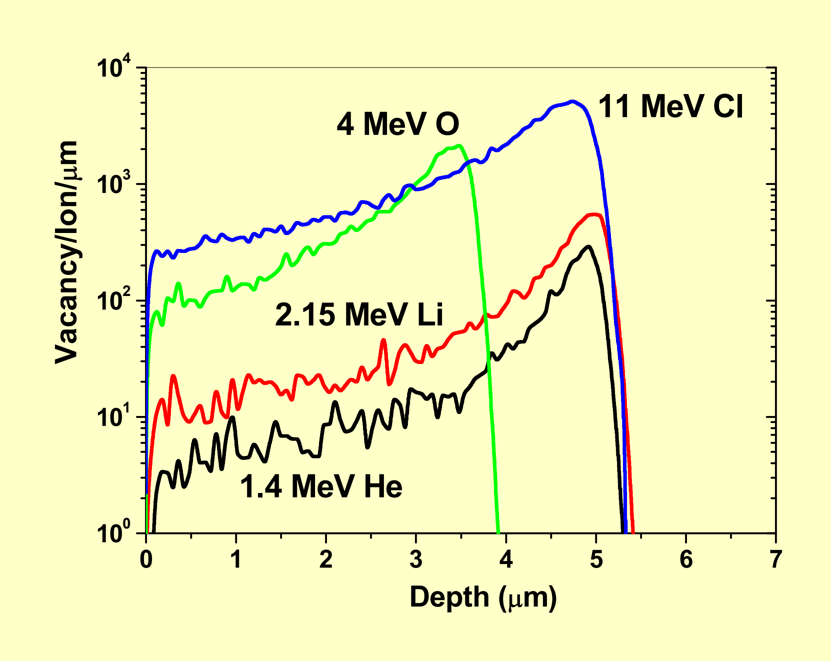
Vacancy profile (from SRIM)

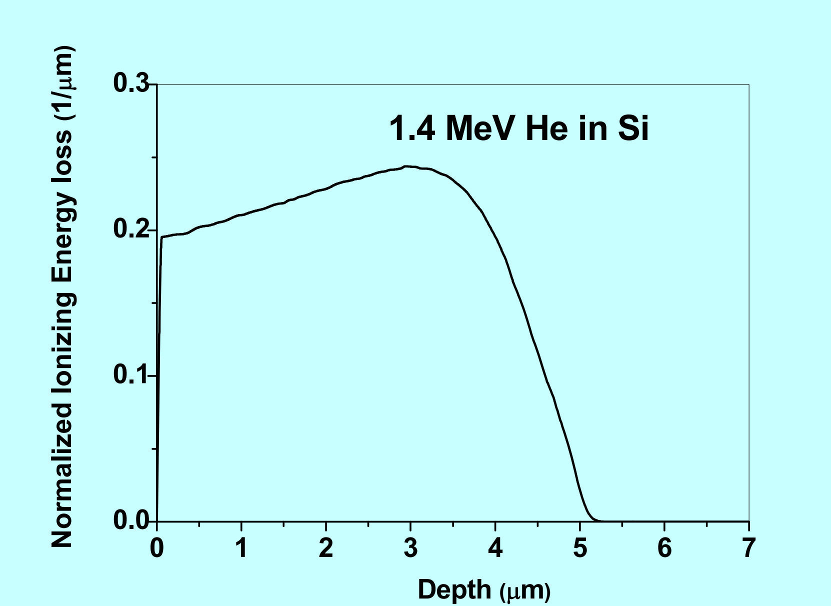
effective capture cross section (from DLTS)

Thermal velocity ($\approx 10^7$ cm/s)

Ion fluence







Data from SRIM

● h+ ● e-

Ramo Theorem
(no diffusion)

Induced charge

from the motion of electrons

$$\eta(x,\Phi) = \frac{1}{w} \cdot \int_x^w dy \cdot \exp\left[-\int_x^y \frac{dz}{v_e(z) \cdot \tau_e(z,\Phi)} \right]$$

from the motion of holes

$$\eta(x,\Phi) = \frac{1}{w} \cdot \int_0^x dy \cdot \exp\left[-\int_y^x \frac{dz}{v_h(z) \cdot \tau_h(z,\Phi)} \right]$$

$$\text{Drift Length} = v_h(z) \cdot \tau_h(z,\Phi) >> w$$

Low damage level

Linearization

$$\eta(x,\Phi) \cong 1 - \Phi \cdot \left[\frac{k_e \cdot \sigma_e \cdot v_{th}}{w} \cdot \int_x^w dy \cdot \int_x^y \frac{Vac(z)}{v_e(z)} dz + \frac{k_h \cdot \sigma_h \cdot v_{th}}{w} \cdot \int_0^x dy \cdot \int_y^x \frac{Vac(z)}{v_h(z)} dz \right]$$

At high bias voltage
Hole contribution negligible
Saturation drift velocity
Semi-analytical expression

$$\mathrm{CCE}(\Phi) = 1 - k_e \cdot \sigma_e \cdot \frac{v_{th}}{\langle v_e \rangle} \cdot \left\{ \int_0^w dz \cdot \left[\tilde{E}_{Ion}(z) \cdot \mathrm{Vac}(z) \cdot \left(1 - \frac{z}{w} \right) \right] \right\} \cdot \Phi \equiv 1 - K_e^* \cdot \Phi_e^*$$

Ion probe energy loss

Vacancy profile

Weighting potential

$$\Phi^* = \text{Effective Fluence} = \int_0^w dz \cdot \left[\tilde{E}_{Ion}(z) \cdot \mathrm{Vac}(z) \cdot \left(1 - \frac{z}{w} \right) \right]$$

$$K_e^* = \text{effective damage factor} = k_e \cdot \sigma_e \cdot \frac{v_{th}}{\langle v_e \rangle}$$

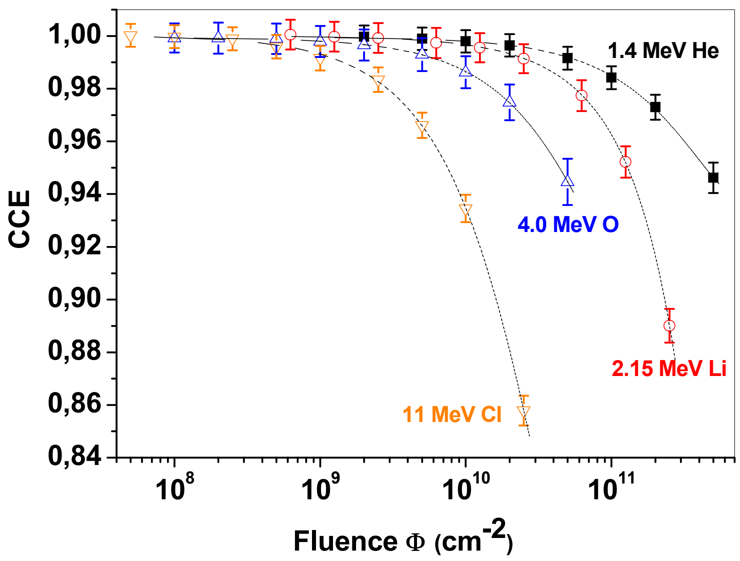
Average drift velocity

Average number of active trap per vacancy

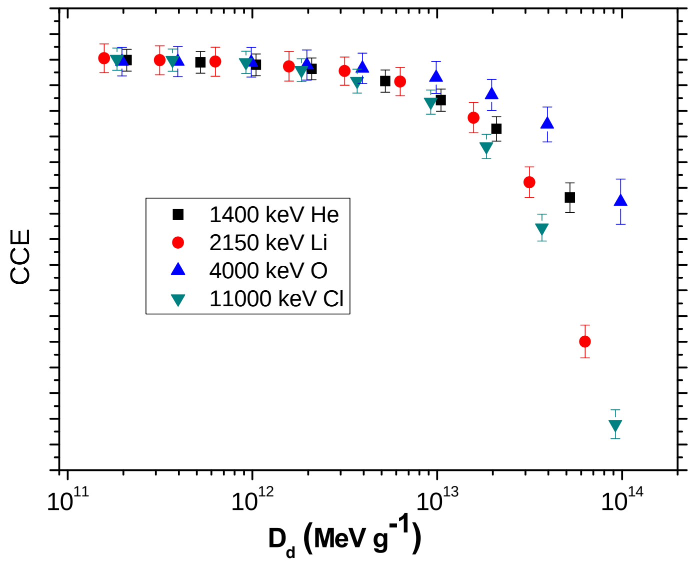
capture cross section of ion induced traps

V_{bias}=100 V

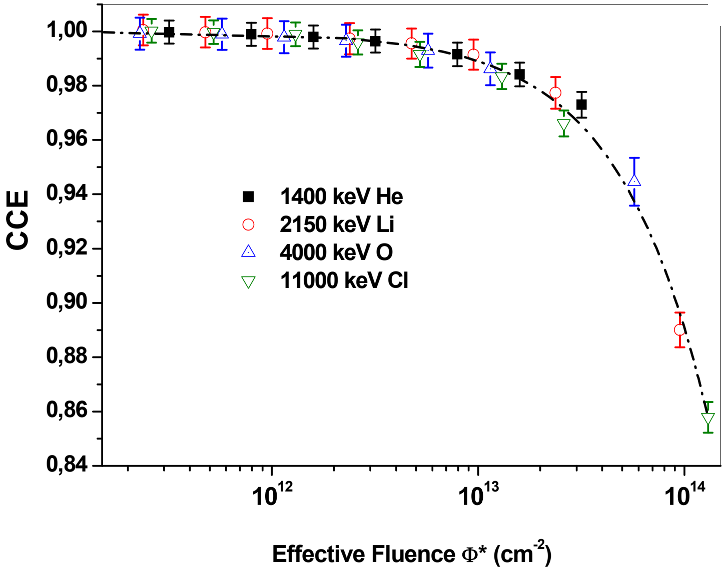
CCE vs Φ

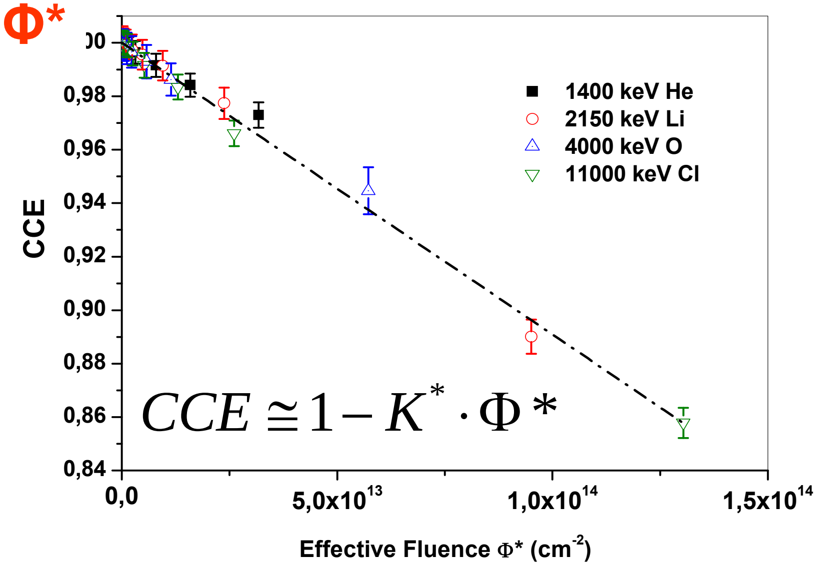


CCE vs D_d

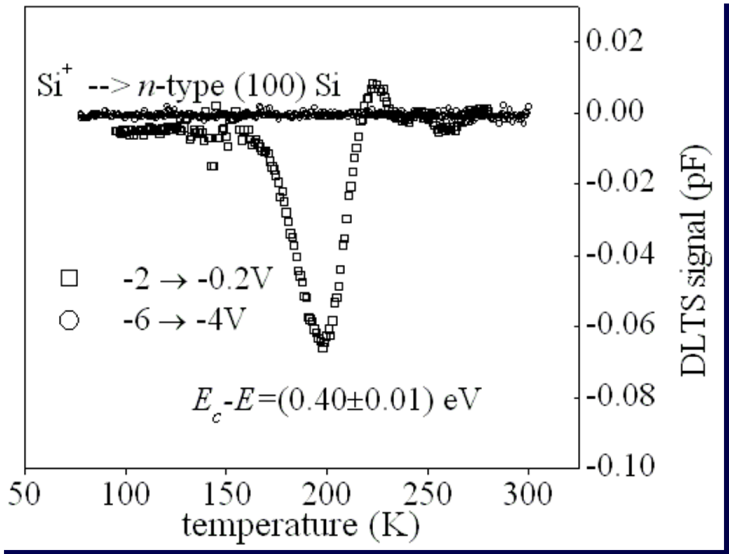


CCE vs Φ*





$$CCE \cong 1 - K^{*} \cdot \Phi^{*}$$



DLTS measurements
singly V2(−/0) negatively charged divacanc

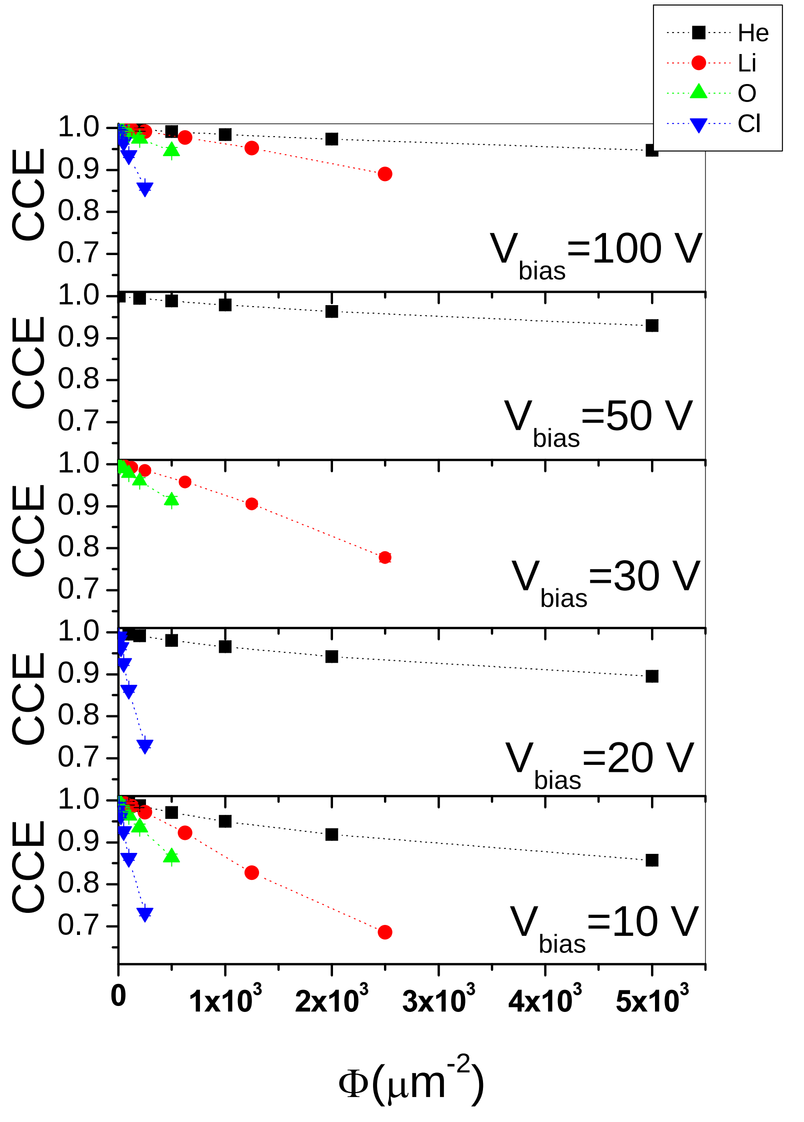
$\sigma_e \approx 5\cdot10^{-15}$ cm^2

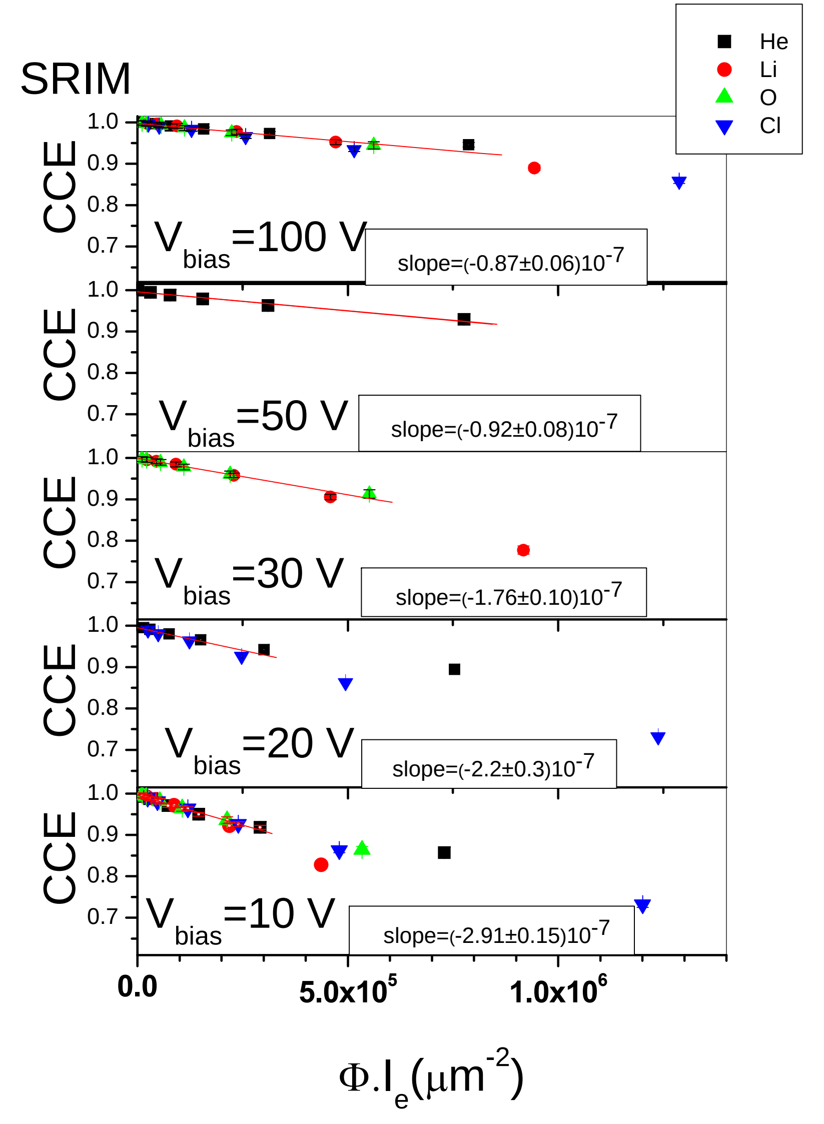
$$K^* = \frac{k_e \cdot \sigma_e \cdot v_{th}}{v_e} = (1.09 \pm 0.02)\cdot10^{-15}\ \mathrm{cm}^2.$$

$k_e \approx 0.2$

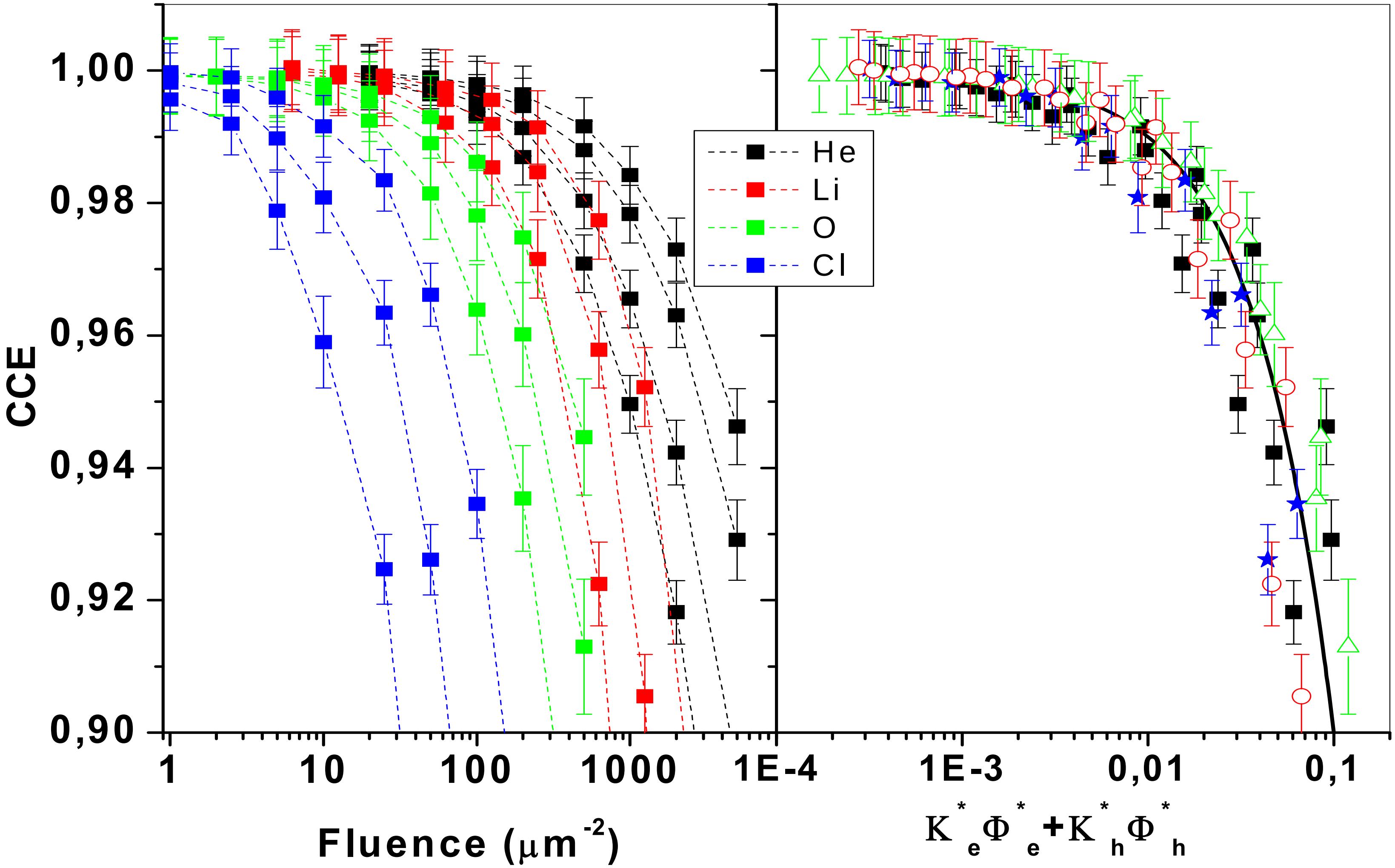
i.e. 5 vacancy to generate an electrically stable trap in low doped n-type silicon

The K* value is independent from the type and energy of the damaging and probing ions and is attributable only to the intrinsic radiation hardness of the material


■ He
● Li
▲ O
▼ Cl
CCE
1.0
0.9
0.8
0.7
V_{bias}=100 V
V_{bias}=50 V
V_{bias}=30 V
V_{bias}=20 V
V_{bias}=10 V
0
$1x10^3$
$2x10^3$
$3x10^3$
$4x10^3$
$5x10^3$
$\Phi(\mu m^{-2})$


SRIM
■ He
● Li
▲ O
▼ Cl
CCE
1.0
0.9
0.8
0.7
V_{bias}=100 V
slope=(-0.87±0.06)10^{-7}
V_{bias}=50 V
slope=(-0.92±0.08)10^{-7}
V_{bias}=30 V
slope=(-1.76±0.10)10^{-7}
V_{bias}=20 V
slope=(-2.2±0.3)10^{-7}
V_{bias}=10 V
slope=(-2.91±0.15)10^{-7}
0.0
$5.0x10^5$
$1.0x10^6$
$\Phi.I_e(\mu m^{-2})$

At different bias voltages



In the low damage regime

The degradation of the CCE of a semiconductor detector due to the damage induced by ions of different mass and energy can be interpreted on the basis of a simplified theory of the IBIC technique.

$$\mathrm{CCE}(\Phi) \equiv 1 - K_e^* \cdot \Phi_e^*$$

Effective fluence

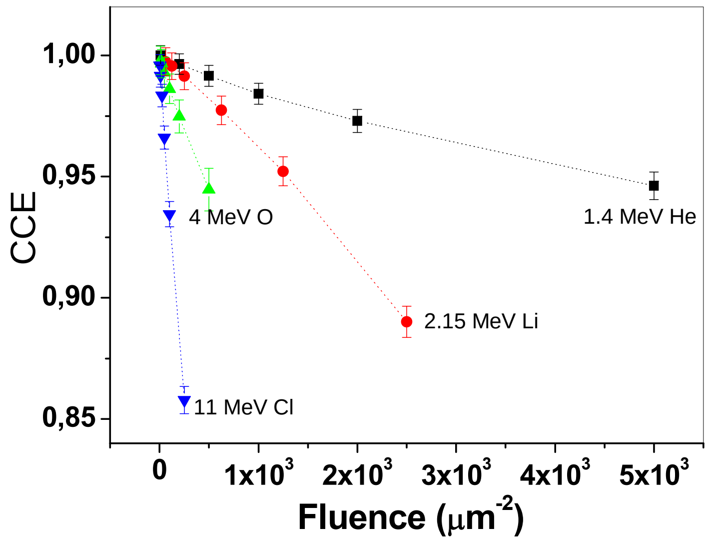
$$\Phi^* = \Phi \cdot \left\{ \frac{1}{d} \cdot \frac{1}{E_P} \int_0^{R_P} dx \frac{dE_P}{dx} \cdot \left[\int_x^d dz \left[V(z) \cdot (d - z) \right] \right] \right\}$$

can be numerically calculated from the vacancy and ionization profiles extracted from the SRIM code.

Effective damage factor

$$K^* = \frac{k_e \cdot \sigma_e \cdot v_{th}}{v_e} = (1.09 \pm 0.02) \cdot 10^{-15}\ \mathrm{cm}^2.$$

the effective damage factor K* is the slope of the CCE degradation as function of Φ^* is proportional to the fraction of the electrically active trap per vacancy



K* can be considered an index that would reliably rank the relative radiation hardness of semiconductors in order to optimize the selection procedure for devices working in high radiation environment.

Effective damage factor

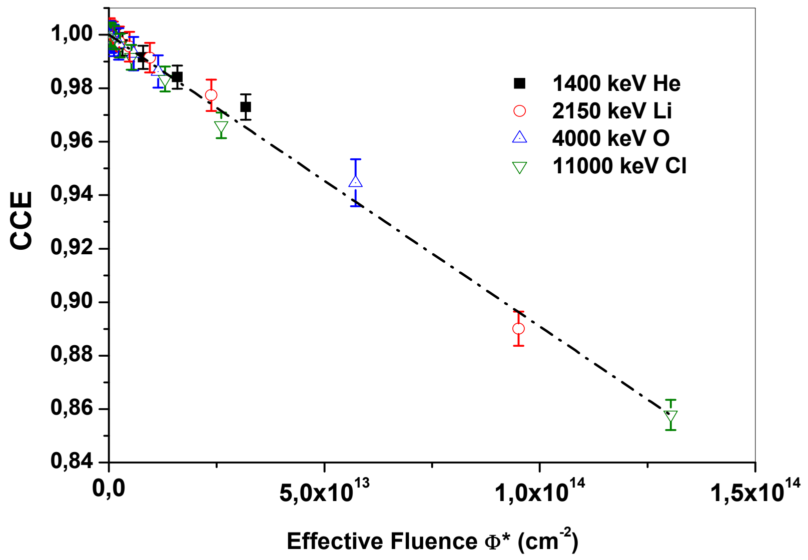
$$K^{*} = \frac{k_e \cdot \sigma_e \cdot v_{th}}{v_e} = (1.09 \pm 0.02)\cdot 10^{-15}\ \mathrm{cm}^2.$$

σ : measured from DLTS

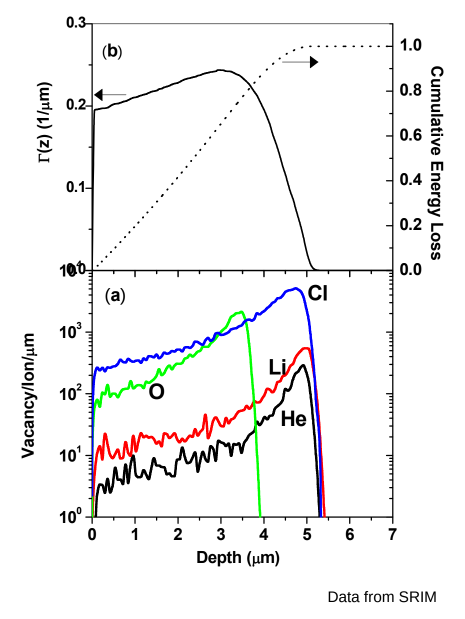
vth: thermal velocity

ve=: electron average velocity

ke: average number of trap/vacancy

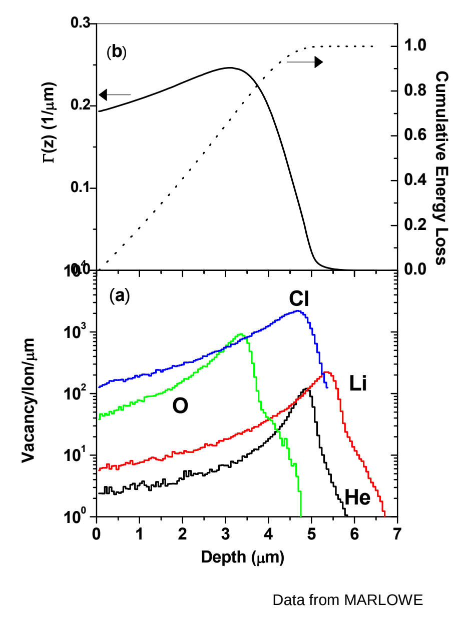


Approach more efficient to condense the CCE degradation data into a single curve than the phenomenological displacement damage dose analysis;
NIEL is valid only in the case of constant vacancy profile.



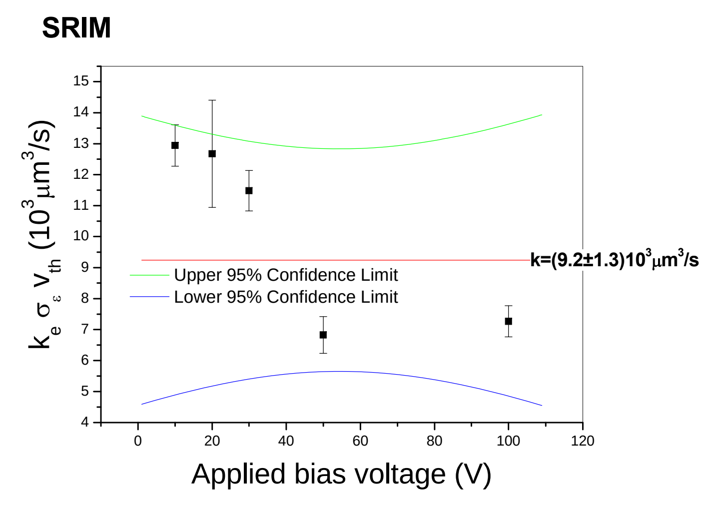
Data from SRIM

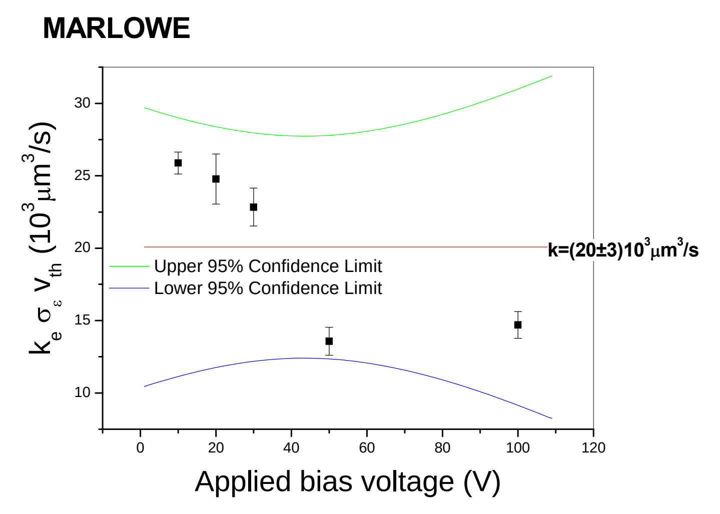
SRIM



Data from MARLOWE

MARLOWE





IAEA Coordinate Research Programme (CRP) F11016 (2011-2015)
“Utilization of ion accelerators for studying and modeling of radiation induced defects in semiconductors and insulators”

Overall Objective:

Use of ion accelerators for improved understanding of how radiation induced defects influence the electronic properties of semiconductor/insulator materials, leading to better understanding of how they degrade or improve the performances of devices in extreme and harsh radiation environments.

Specific Research Objective:

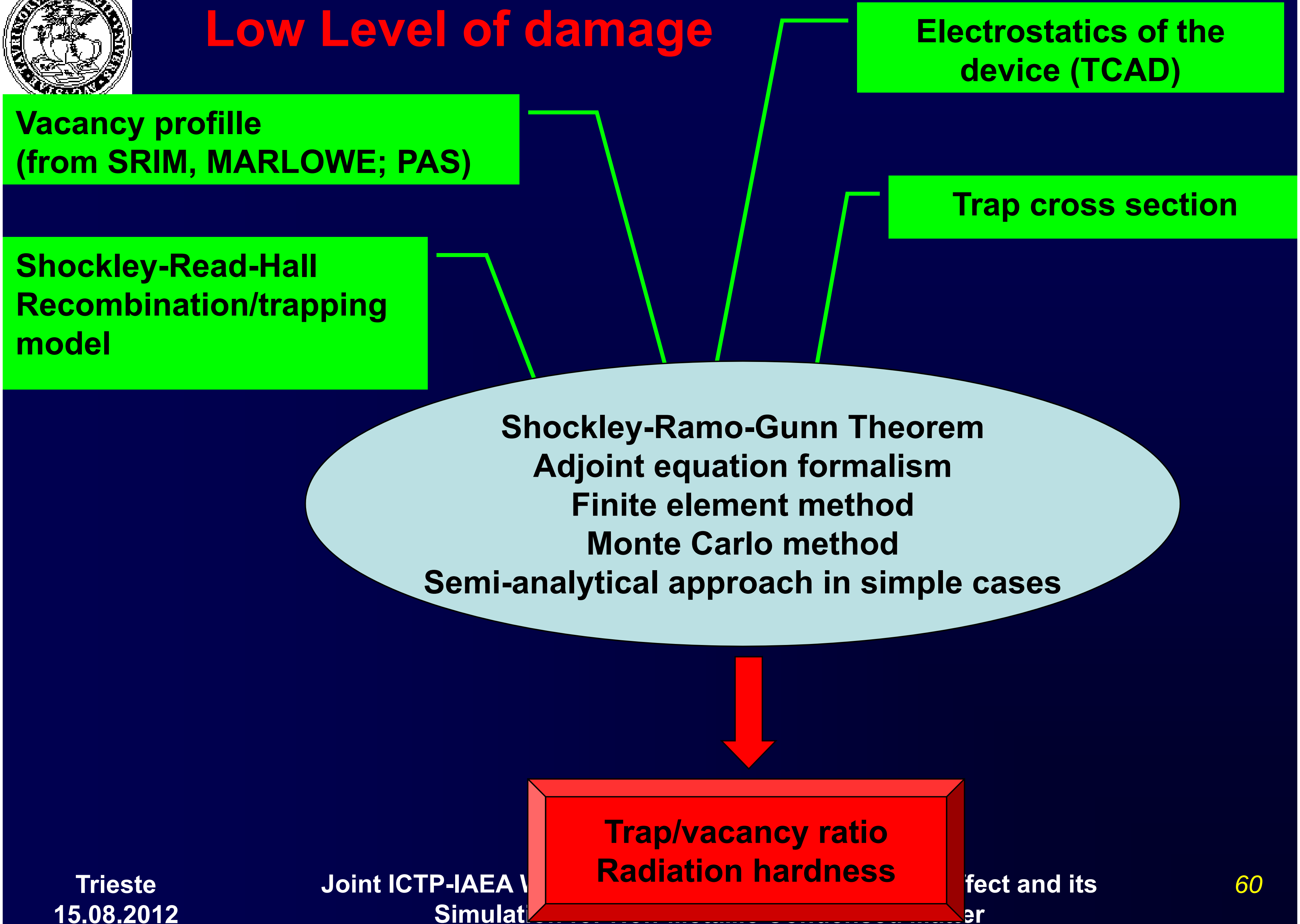
Deeper theoretical knowledge and experimental data on defects created by light and heavy ions; in terms of their type, density and effect on fundamental electronic properties of semiconductors and insulators.

Expected Research Outputs:

Definition of an experimental protocol to determine the key parameters for the characterization of the effects of radiation damage on semiconductor materials and devices.

Refined theoretical models for defect generation and for modelling their effect on electronic properties.

Low Level of damage



Electrostatics of the device (TCAD)

Vacancy profille (from SRIM, MARLOWE; PAS)

Trap cross section

Shockley-Read-Hall Recombination/trapping model

Shockley-Ramo-Gunn Theorem
Adjoint equation formalism
Finite element method
Monte Carlo method
Semi-analytical approach in simple cases

Trap/vacancy ratio
Radiation hardness